ABSTRACTS

FORTH LATIN-AMERICAN WORKSHOP ON CLIQUES IN GRAPHS

NOVEMBER 16-19, 2010

FORTH LATIN-AMERICAN WORKSHOP ON CLIQUES IN GRAPHS

UNIVERSIDADE FEDERAL DO RIO DE JANEIRO
NOVEMBER 16-19, 2010
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The workshop is meant to foster interaction between the latin american graph theory and combinatory researchers, whose research interests include cliques, clique graphs, and the behavior of cliques and related issues. The official languages are English, Portuguese and Spanish. This is the forth edition of this workshop, after:

- Workshop Latino-Americano de Cliques em Grafos
  April 17th - 19th, 2002.
  Rio de Janeiro, Brasil.
- Second Latin-American Workshop on Cliques in Graphs
  October 18th-20th, 2006.
  La Plata, Argentina.
- Tercer Taller Latinoamericano de Clanes en Graficas
  October 28th-31st, 2008.
  Guanajuato, Mexico

During the meeting 32 scientific communications will be exposed and there will be 4 plenary talks. We are very grateful to all the participants for their contributions and particularly to the invited speakers for their interest.

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Celina Miraglia Herrera de Figueiredo
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List of Plenary Conferences

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On Independent Vertex Sets and Variants of Matchings

Andreas Brandstädt
Department of Computer Science, University of Rostock, Germany

(joint work with Vassilis Giakoumakis, Chinh T. Hoàng, Van Bang Le, Vadim V. Lozin, Raffaele Mosca and Ragnar Nevries)

In a finite undirected graph, a vertex set is *independent* (or *stable*) if its vertices are pairwise nonadjacent. For given graph G, the Maximum Independent Set (MIS) Problem asks for an independent vertex set of maximum size in G. The MWIS problem is the vertex-weighted version of MIS.

It is well known that clique separator decomposition and modular decomposition are helpful tools for solving the MWIS problem. One of our results allows to combine both of them. This implies various improvements of known results, among them a polynomial time algorithm for MWIS on apple-free graphs–a common generalization of chordal graphs, cographs and claw-free graphs. We also discuss how to solve efficiently the MWIS problem on some subclasses of hole-free graphs and of P_5-free graphs.

A distance-k matching in graph G is a subset of edges whose pairwise distance is at least k. Thus, the well-known notion of an induced matching is the same as distance-2 matching. Obviously, a distance-k matching in G is an independent vertex set in the k-th power $L(G)^k$ of the line graph $L(G)$ of G and vice versa. For given G, the Maximum Induced Matching (MIM) Problem asks for an independent vertex set in the square $L(G)^2$ of $L(G)$. Unlike the problem Maximum Matching, the MIM problem is NP-complete even on very restricted bipartite graphs and on claw-free graphs. Many papers are dealing with the complexity of the MIM problem on particular graph classes. We discuss the complexity of this problem and its generalization in $L(G)^k$ for $k \geq 3$ for some important graph classes.

The Dominating Induced Matching (DIM) Problem (also called Efficient Edge Domination (EED) Problem in various papers) asks for the existence of an independent vertex set in $L(G)^2$ which simultaneously is a dominating set in $L(G)$. We show that this problem is solvable in polynomial time for hole-free graphs.

Partitions of direct products of complete graphs into independent dominating sets

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Keywords: Independent dominating sets, direct product graphs, complete graphs.

Let $G = (V, E)$ be an undirected finite simple graph without loops. A set $S \subseteq V$ is called a *dominating set* if for every vertex $v \in V \setminus S$ there exists a vertex $u \in S$ such that u is adjacent to v. A set $S \subseteq V$ is called *independent* if no two vertices in S are adjacent. A set $S \subseteq V$ is called an *independent dominating set* of G if it is both independent and dominating set of G. A partition of the vertex set V into independent dominating sets is called an *idomatic partition* of G. Clearly, an idomatic partition of a graph G represents a proper coloring of the vertices of G. The maximum order of an idomatic partition of G is called the *idomatic number* $id(G)$ and this parameter was introduced by Cockayne and Hedetniemi in 1977. Notice that not every graph has an idomatic partition. For example, C_5 has no idomatic partition. The *direct product* $G \times H$ of two graphs G and H is defined by $V(G \times H) = V(G) \times V(H)$, and where two vertices $(u_1, u_2), (v_1, v_2)$ are joined by an edge in $E(G \times H)$ if $\{u_1, v_1\} \in E(G)$ and $\{u_2, v_2\} \in E(H)$.

In this talk, we give a full characterization of the idomatic partitions of the direct product of three complete graphs by using an elementary algebraic approach. This partially answer a question of Dunbar et al. 2000.

Orientations of graphs

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In this survey talk I will present results on orientations of graphs satisfying connectivity and/or parity constraints. I will provide interesting applications, connections between the results, open problems and nice conjectures.

New results on Dually Chordal Graphs

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Abstract

Dually Chordal Graphs appear independently in 3 articles with 3 different definitions and 3 different names [1, 2, 3]. Today it is very clear for us that dually chordal is the most appropriate because this class is dual in many ways to that of chordal graphs.

In this talk we shall give the new results that we have obtained about dually chordal graphs.

It is known that the simplicial vertices realize the eccentricity and the diameter in chordal graphs. We will show that this role is played by vertices with maximum neighbor in dually chordal graphs and other special vertices in related classes.

Dually chordal graphs can be characterized as those graphs owning an spanning tree, called compatible tree, such that the family of cliques (or the family of neighborhoods) induces a family of subtrees. We prove that the family of all the minimal separators or any subfamily obtained considering a minimal separator for each pair of non adjacent vertices can be used to obtain similar characterizations.

Other property of dually chordal graphs related to cliques is that a graph is dually chordal if and only if it is clique Helly and its clique graph is chordal. We prove that the family of cliques can be replaced by minimal separators in this context also.

Finally, by using the fact that the clique graph of a chordal graph is a dually chordal graph we explore the relationship between the clique trees of a chordal graph and the compatible trees of its clique graph. These results allowed us answer if, given a set of vertices of a dually chordal graph, there is a compatible tree that has this set as its leaves.

*In cooperation with Pablo De Caria.

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Improved Algorithms for Minimum Clique-Transversal Problem

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Keywords: clique-transversal, algorithms, $\overline{3K_2}$-free circular-arc graphs.

A clique-transversal of a graph G is a subset of vertices intersecting all the cliques of G. It is NP-hard to determine the minimum cardinality $\tau_c(G)$ of a clique-transversal of G. Durán, Lin, Mera and Szwarcfiter [Ann Oper Res 157 (2008), pp. 37-45], proposed algorithms for determining this parameter for several graph classes: (i) one for general graphs, which runs in $O(n^{2\tau_c(G)})$ time; (ii) one for $\overline{3K_2}$-free circular-arc graphs in $O(n^4)$ time; (iii) one for Helly circular-arc graphs in $O(n)$ time. In present work, we improved the time-complexities of (i) and (ii): an algorithm of $O(n^{\tau_c(G)-1}m^{\frac{\tau_c(G)}{2}})$ for general graphs and an $O(n)$-time algorithm for $\overline{3K_2}$-free circular-arc graphs. Also, we give an algorithm of $O(\tau_c(G) \cdot max\{\tau_c(G), \sqrt{m}\} \cdot nm^{\frac{\tau_c(G)-2}{2}})$ time for dually-chordal graphs.

Biclique Transversal and Biclique Independent Set

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Keywords: Biclique, transversals, independent set

A *biclique* of a graph G is a maximal complete bipartite induced subgraph of G.

A *biclique-transversal set* of a graph G is a set of vertices intersecting all bicliques of G. The biclique-transversal number $\tau_b(G)$ is the cardinality of the minimum biclique-transversal set of G. A *minimum biclique-transversal set* is a biclique-transversal set of cardinality $\tau_b(G)$.

A *biclique-independent set* of a graph G is a collection of vertex-disjoint bicliques of G. The biclique-independent number $\alpha_b(G)$ is the cardinality of the maximum biclique-independent set of G. A *maximum biclique-independent set* is a biclique-independent set of cardinality $\alpha_b(G)$.

For any graph, it follows that $\alpha_b(G) \leq \tau_b(G)$. When $\alpha_b(H) = \tau_b(H)$ for every induced subgraph H of G, we say that G is *biclique-perfect*.

In this work we study the computational complexity of finding $\tau_b(G)$ and $\alpha_b(G)$. We prove that the problem of finding the minimum biclique-transversal is NP-Hard and the maximum-independent set is NP-Complete for the general case.

We also study these problem by restricting to some classes of graphs. We present some classes for which finding α_b and τ_b is NP-Complete and also classes where these problems are polynomially solvable.

Finally, we present some biclique-perfect classes of graphs and some minimal biclique-imperfect graphs, and show that the difference between α_b and τ_b can be arbitrarily large.

Algorithmic aspects of Steiner convexity and enumeration of Steiner trees

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Keywords: convex set, steiner tree, steiner interval.

In this work we study the algorithmic complexity of deciding if a vertex x is in some W-Steiner tree, that is, if x is a candidate to be a Steiner vertex. We also show how this problem is used to define a notion of Steiner convexity in graphs. We design an algorithm for the enumeration of all Steiner trees for a bounded number of terminals, which is the usual scenario in many applications. We discuss algorithmic issues involving space requirements to represent the optimal solutions and the time delay to generate them. Our algorithm generates all W-Steiner trees with $O(n)$ delay and $O(\alpha)$ space, where $n = |V(G)|$ and α is the number of W-Steiner trees.

Alliances and Graph Convexity

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Keywords: alliance, convexity, hull set, max-snp, approximation

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Let $G = (V, E)$ be a graph, a set $S \subset V$ is an *alliance* of G if for each vertex $v \in S$, v and its neighbors in S are not outnumbered by the neighbors of v in $\overline{S} = V \setminus S$, or $|N[v] \cap S| \geq |N(v) \cap \overline{S}|$, and $\overline{S}$ is called a *co-alliance* of G. The *anti-alliance interval* $I_a(S)$ of a set $S \subseteq V$ is the set of the vertices of S plus the vertices v of $\overline{S}$ having more neighbors in S than in $\overline{S}$ including v or $I_a(S) = S \cup \{v \in \overline{S}$, such that $|N(v) \cap S| > |N[v] \cap \overline{S}|\}$. The set S is an *anti-alliance set* of G if $I_a(S) = V$, since no alliance is contained in $V \setminus S$. The size of the smallest anti-alliance set of G is the *anti-alliance number* of G, denoted by $n_a(G)$. The size of the largest proper co-alliance $c_a(G)$ of G is the *anti-alliance convexity number* of G. The size of the smallest co-alliance of G containing S is the *anti-alliance hull* of S, denoted by $H_a(S)$. If $H_a(S) = V(G)$ then S is an *anti-alliance hull* of G, and if S is minimum then $h_a(G) = |S|$ is the *anti-alliance hull number* of G. We prove that computing $n_a(G)$ and $h_a(G)$ are Max SNP-hard problems for planar cubic graphs. It is known that determining $c_a(G)$ is NP-hard for split graphs. Given an ε–approximation for a minimum vertex cover of $G = (V, E)$ we define in polynomial time an $\varepsilon\frac{2\Delta+\delta+2}{\delta+2}$–approximation for $n_a(G)$ of $G = (V, E)$, where Δ and δ are the maximum and the minimum degrees of G. If $G = (V, E)$ is a d-regular graph, we prove that the problem of determining $c_a(G)$ is polynomial when $d \leq 5$, and that given a 6-regular graph $G = (V, E)$ and a fixed vertex $v \in V$ it is NP-complete to decide whether there is an alliance S of G containing v with size $|S| \leq k$. We prove that it is a polynomial problem to determine the anti-alliance number when $G = (V, E)$ is a tree.

Convex Covers of Graphs[1]

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Keywords: Convexity, covering of graphs, partition of graphs.

We consider only finite, simple graphs. Let G be a graph and $S \subseteq V(G)$, its *closed interval* $I[S]$ is the set of vertices lying on shortest paths between any pair of vertices of S. The set S is *convex* if $I[S] = S$. In this work we define the concept of convex cover of graphs. A graph G has a *convex p-cover* if $V(G)$ can be covered by p convex sets, i.e., there exists $\mathcal{V} = (V_1, \ldots, V_p)$, $p \in \mathbb{N}$, such that $V(G) = \bigcup_{1 \le i \le p} V_i$; for $1 \le i \le p$, V_i is convex and $V_i \not\subseteq \bigcup_{\substack{1 \le i \le p \\ i \ne j}} V_j$.

The concept of convex cover is related to convex partition which was proposed in [1]. The convex p-partition of a graph is a particular case of convex cover where all the sets of $\mathcal{V}$ are disjoints. In a different context, A. Prisacaru considered, in his PhD. Thesis, the problem of covering a set of vertices by convex sets.

In this work, we prove that it is NP-complete to decide if a graph G has a convex p-cover for a fixed integer $p \ge 3$. We show that all connected chordal graphs have a convex p-cover, for any integer $1 \le p \le n$. We also establish conditions on n and k to decide if a power of cycle has a convex p-cover. Finally, we develop an algorithm for disconnected graphs.

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Biclique-Coloring

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Keywords: bicliques, computational complexity, graph coloring, biclique coloring.

A k-clique-coloring of a graph is an assignment of k colors to its vertices such that every clique has at least two vertices with different colors. For $k \geq 2$, the problem of deciding if a graph is k-clique-colorable is Σ_2^p-complete, though it is easier for some graph classes.

In this work, we study the computational complexity of the k-biclique-coloring problem, which we define as the analogue of the k-clique-coloring for bicliques. That is, a k-biclique-coloring of a graph is an assignment of k colors to its vertices such that every biclique has at least two vertices with different colors.

We prove that the k-biclique-coloring problem is Σ_2^p-complete for $k \geq 2$, even for $K_{3,3}$-free graphs, and show that it is NP-Complete for $k \geq 2$ on the class of split graphs and for $k = 2$ on the class of $(W_4, dart, gem)$-free graphs. Also, we show that it can be solved in polynomial time for threshold graphs, block graphs and graphs in which every edge belongs to a single biclique.

Clique-perfectness of complements of line graphs[2]

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Keywords: clique-perfect graphs, complements of line graphs, perfect graphs

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A *clique-transversal* of a graph G is a subset of vertices that meets all the maximal cliques of G. A *clique-independent set* is a collection of pairwise vertex-disjoint maximal cliques. The *clique-transversal number* and *clique-independence number* of G are the sizes of a minimum clique-transversal and a maximum clique-independent set of G, respectively. A graph G is *clique-perfect* if these two numbers are equal for every induced subgraph of G. Unlike the class of perfect graphs, the class of clique-perfect graphs is not closed under complementation neither a characterization by minimal forbidden induced subgraphs is known. Nevertheless, partial results in this direction have been obtained; i.e., characterizations of clique-perfect graphs by a restricted list of forbidden induced subgraphs when the graph is known to belong to certain graph classes. For instance, in [1], a characterization of those line graphs that are clique-perfect is given in terms of minimal forbidden induced subgraphs. We studied clique-perfectness of complements of line graphs. Precisely, the main result of this work is a characterization of the clique-perfectness of all complements of line graphs in terms of minimal forbidden induced subgraphs.

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In path graphs there is a path model that realizes the leafage

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Keywords: path graphs, clique trees, leafage.

Chordal graphs were defined as those graphs which have not induced cycles of 4 or more vertices. Subsequently, Gavril [1] prove that a graph G is chordal if and only if it has a clique tree, i. e. a tree T whose vertices are cliques of G and such that for every vertex x of G, C_x, clique of G containing x, induces a subtree of T.

Interval and path graph(UV) graphs are subclasses of chordal graph and they can be characterized by clique trees. A graph G is a path graph(UV) if there exists T a tree such that every C_x induces a path in T, it is called path model of G. Clearly $Interval \subset UV$.

The leafage of a chordal graph [2], denoted $l(G)$ is the minimun number of leaves of its clique trees. A clique tree T of G realizes the leafage if the number of leaves of T is exactly $l(G)$.

Clearly, if G is an *Interval* graph, there is T a clique tree of G that is a path that realizes the leafage, i.e the number of leaves of T is $l(G)$. By the before exposed, it is natural to ask: if G is a path graph then is there a path model that realizes the leafage? In this work, we answer this question.

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An evidence for Lovász conjecture about Hamiltonian paths and cycles

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Keywords: Cayley graphs, Hamiltonian cycle, Lovász conjecture, graph $H_{l,p}$.

It was conjectured by Lovász that every connected vertex-transitive graph has a Hamiltonian path [1]. So far, only four connected vertex-transitive graphs with more than two vertices but without Hamiltonian cycles are known [2]. These four graphs have Hamiltonian paths. However, since none of these four graphs is a Cayley graph, we can look at the Lovász conjecture as stating that every connected Cayley graph with more than two vertices has a Hamiltonian cycle.

In this work, we show some properties of the gadget graph $H_{l,p}$ which was used by Holyer to prove the $\mathcal{NP}-$completeness of the edge partition into cliques problem [3]. We show that this graph is a Cayley graph and present a construction of a Hamiltonian cycle, which corroborates the Lovász conjecture.

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A necessary condition for EPT graphs

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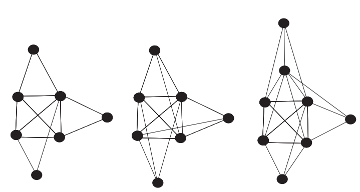
Keywords: intersection graphs, EPT graphs, forbidden subgraphs.

An (h, s, t)-representation of a graph G consists of a collection of subtrees of a tree T, where each subtree corresponds to a vertex in G, such that (i) the maximum degree of T is at most h, (ii) every subtree has maximum degree at most s, (iii) there is an edge between two vertices in the graphs G if and only if the corresponding subtrees have at least t vertex in common in T. The class of graphs that have an (h, s, t)-representation is denoted by $[h, s, t]$. It is known that $[3, 2, 2]$ is the class of chordal EPT-graphs and $[4, 2, 2]$ is the class of weakly chordal EPT-graphs. We want to find a list of forbidden subgraps for the $(3, 2, 2)$-graphs.

Let C be a clique of G. A vertex v of G is a satellite of C if $B_v = N(v) \cap C$ is a non-empty proper subset of C. B_v is called the base of v and it is said minimal if no other base of a satellite of C is properly contained in B_v. In this work we prove the following theorem:

Theorem: Let C be a clique of an EPT graph G. If $w \in C$ then w belongs to at most two minimal bases of satellites of C.

We characterize the minimal graphs which do not satisfy the condition of the previous theorem, as a consequence we present a finite family of forbidden subgraphs for the class EPT and thus for the class [3,2,2].



S1 S2 S3

Figure 1: Forbidden induced subgraph for EPT-graphs

Results on Determining the Minimum Number of Lengths in Interval Models

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Keywords: Interval count; interval graphs; interval lengths.

Many structural properties of interval graphs have been studied and NP-complete problems for general graphs have been efficiently solved for this class of graphs. However, a basic problem related to them remains open despite the effort devoted.

Given an interval graph G, the *interval count problem* is that of determining the smallest number $IC(G)$ of interval lengths needed to represent an interval model of G. The question of deciding whether $IC(G) = 1$ is equivalent to that of recognizing whether G is a unit-interval graph, a problem for which well-known linear-time algorithms exist. On the other hand, it is neither known whether the complexity of deciding if $IC(G) = k$ is polynomially-time solvable, nor if the problem is NP-complete, for any fixed $k \geq 2$. Restricted to graph classes, the efficient computation of interval counts is known only for trees, threshold graphs, almost-$K_{1,3}$-free graphs, and starlike-threshold graphs.

In this work, we provide a short survey on the interval count and related problems. We present polynomial-time algorithms to compute the interval count of generalized-threshold graphs and extended-bull-free graphs (a generalization of trivially perfect graphs), the latter class containing instances of graphs with arbitrary interval count values. To our knowledge, there are no other subclasses of interval graphs for which it is currently known how to compute their interval counts efficiently.

New Results on Total Coloring of Snarks

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Keywords: snarks, total coloring, dot product, star product, square product

Snarks are cubic bridgeless graphs of chromatic index 4 which had their origin in the search of counterexamples to the Four Color Theorem. In 2003, Cavicchioli et al. [2] proved that for snarks with less than 30 vertices, the total chromatic number is 4, and proposed the problem of finding (if any) the smallest snark which is not 4-total colorable. The only known families of snarks that had their total chromatic number determined to be 4 were the Flower Snark family and the Goldberg family [1].

Recently, we proved that the Loupekhine family and the Blanusa families are 4-total colorable. We constructed additional snark families using the *dot product* and we determined that their total chromatic number is 4 [3].

The 4-total coloring of Blanusa families uses two fixed 4-total colorings of the Petersen graph without two adjacent vertices. In this work, we prove that it is not possible to use only one fixed 4-total coloring for this graph. Moreover, we show properties about 4-total coloring of the *square product* and the *star product*, operations to construct snarks. Finally, we prove that the star product between Loupekhine, Goldberg and Flower snarks are 4-total colorable.

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Adjacent vertex-distinguishing total coloring of indifference graphs

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Keywords: Total coloring, indifference graphs.

The problem discussed in this work is the Adjacent Vertex-Distinguishing Total Coloring (ADVTC) of graphs, which asks for the minimum proper total coloring of a graph with an additional property: if two vertices are adjacent in a graph, then there must be at least one color that is used by one of them and not used by the other. The meaning of used in this context is that a color is used by a vertex if it is the color of that vertex or the color of some edge incident to that vertex.

This problem was introduced by Zhang et al. [1], that denoted as $\chi_{at}(G)$ the minimum number of colors of an ADVTC of a graph G. They solved the problem for cycles, complete graphs, fans, complete bipartite graphs, paths, and trees. Moreover, they conjectured that $\chi_{at}(G) \leq \Delta(G) + 3$. Later, Chen [2] showed that this conjecture holds for graphs with $\Delta(G) = 3$.

Indifference graphs are graphs whose vertices can be linearly ordered in such a way that the vertices of any maximal clique appear consecutively in the order. The problems of edge and total coloring for these graphs have been solved only under special conditions.

In this work, we show that the conjecture of Zhang et al. holds for indifference graphs. We show also optimal ADVTC for indifference graphs with odd maximum degree and at least two adjacent maximum-degree vertices.

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On $L(2,1)$-coloring split permutation graphs

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Keywords: $L(2,1)$-coloring, split graph, permutation graph.

A k-$L(2,1)$-*coloring* of a graph is an assignment of colors in $\{0,\ldots,k\}$ to its vertices such that adjacent vertices get colors at least two apart, and vertices at distance two get distinct colors. Let $\lambda(G)$ be the minimum value of k such that there exists a k-$L(2,1)$-coloring of G.

The problem of computing $\lambda(G)$ arises in the context of the radio frequency assignment problem and has been widely studied [3]. This problem is $\mathcal{NP}$-hard even when restricted to split graphs [2]. Although there are linear-time algorithms for computing $\lambda(G)$ of bipartite chain graphs [1] and P_4-tidy graphs [4], the problem is still open both for proper interval graphs and bipartite permutation graphs. We show results on computing $\lambda(G)$ when G belongs to the class of split permutation graphs, a subclass of hereditary clique-Helly graphs.

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A note on class 2 split graphs [3]

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Keywords: split graphs, edge coloring, classification problem

The *Classification Problem* is the problem of deciding whether a simple graph has chromatic index equals to Δ or $\Delta + 1$. In the first case, the graphs are called *Classe 1*, otherwise, they are *Class 2*. It is NP-complete to decide if a graph has chromatic index equals to Δ whereas it is co-NP-complete to decide if the chromatic index is equals to $\Delta + 1$.

A simple graph $G = (V, E)$ is *overfull* when $|E| > \Delta \left\lfloor \frac{|V|}{2} \right\rfloor$. If G has an overfull subgraph H with $\Delta(H) = \Delta(G)$, then G is called *subgraph-overfull*. When the overfull subgraph H is induced by a $\Delta(G)$-vertex v and its neighbors, denoted by $N[v]$, then G is a *neighborhood-overfull* graph. Overfull, subgraph-overfull and neighborhood-overfull graphs are Class 2. According to the famous *Overfull Conjecture*, being Class 2 is equivalent to being subgraph-overfull, when the graph has $\Delta(G) \geq \frac{|V(G)|}{3}$.

A *split graph* G is a graph whose vertex set admits a partition (Q, S) into a stable set S and a clique Q. Figueiredo et al. shows that every subgraph-overfull split graph is in fact neighborhood-overfull. In the same article, they conjecture that every Class 2 chordal graph is neighborhood-overfull. It is known that a split graph is a chordal graph.

Let's assume that Q is a maximal clique of a split graph G and let $d(Q) = \Delta(G) - |Q| + 1$. We show that G is neighborhood-overfull if and only if $\Delta(G)$ is even, $d^2(Q) < |Q| - 1$, and G has a minimum number of twin $\Delta(G)$-vertices v such that $G[N[v]]$ is overfull. We also show that for neighborhood-overfull split graphs, $\Delta(G) > \frac{|V(G)|}{3}$. Therefore, for split graphs the above two conjectures are equivalent. If these conjectures are true, our characterization gives the unique Class 2 split graphs.

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On recognizing split clique graphs

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Keywords: clique graphs; split graphs; recognition problems.

The problem of recognizing clique graphs in a graph class A consists in, given any graph $G \in A$, to determine if G is a clique graph. When A is the class of all graphs the problem is NP-complete[4]. When A is contained in the class of Clique Helly graphs the problem is trivial since every clique Helly graph is a clique graph[5].

We are interested in determining a class A, not contained in Clique Helly graphs, for which the problem is polynomial. Up to now, no such a class is know. The present work considers the class of split graphs as a possible such class. As an approach to the matter, we prove the following two theorems.

Let G be a split graph, $V(G) = S \cup K$, S a stable set, K a complete set and $K = \bigcup_{s \in S} N(s)$.

Theorem 1: If $| S | \leq 3$, then G is a clique graph if and only if G is not the Hajós graph.

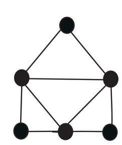

Figure 2: The Hajós graph.

Theorem 2: If no $N(s)$, $s \in S$, is contained in the union of the remaining $N(t)$, $t \in S \setminus \{s\}$, then G is a clique graph.

Notice that one theorem does not imply the other and that they resolve the problem for two subclasses of split graphs not contained in Clique Helly graphs.

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Contributions to the Cluster Editing Problem

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Keywords: Cluster Editing Problem, Combinatorial Optimization, Clusterization

The Cluster Editing Problem is defined as follows: given as input an undirected graph $G = (V, E)$, by adding edges to G and/or removing edges from G, it must be transformed into a cluster graph, that is, an union of disjoint cliques. The cluster editing problem was proved to be NP-complete and models several practical possible applications in the fields of image processing, computational biology and more.

This work focuses on the classic non-weighted cluster editing problem. Integer programming and heuristic techniques are used. Two algorithms for instance generation are presented. A graph-theoretic formulation of the problem is used to exactly solve instances. However, due to the excessive amount of time needed to solve harder instances, two new heuristics are proposed for solving the problem in these cases. As future work, the development of more sofisticated algorithms, like meta-heuristics and hybrid algorithms, are planned in hope of producing better results.

Gene clusters as intersections of powers of paths

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Keywords: power of a path, unit interval graph, genome, gene clusters

There are various definitions of a gene cluster determined by two genomes and methods for finding these clusters. However, there is little work on characterizing configurations of genes that are eligible to be a cluster according to a given definition. For example, in the ancestor of two genomes, which sets of genes could possibly reflect a cluster in the two daughter genomes? i.e. given a set of genes in a genome is it always possible to find two genomes such that their intersection is exactly this cluster? In one version of this problem, we make use of the graph theoretical definition of a cluster in [1].

Let V_X to be the set of n markers in the genome X. These markers are partitioned among a number of total orders called *c*hromosomes. For markers g and h in V_X on the same chromosome in X, let $gh \in E_X$ if the number of genes intervening between g and h in X is less than θ, where $\theta \geq 1$ is a fixed *n*eighbourhood parameter. We call $G_X = (V_X, E_X)$ a *θ-adjacent graph* if its edges are determined by a neighbourhood parameter θ (*powers of paths* P_n^θ).

Then the problem can be reformulated as follows: given a graph G, does there exist two θ-adjacency graphs $G_S = (V_S, E_S)$ and $G_T = (V_T, E_T)$, such that $G \subseteq G_S \cap G_T$, i.e. $V \subseteq V_S \cap V_T \neq \emptyset$ and $E \subseteq E_S \cap E_T$? In this work, we study the case when G is an unit interval graph and we show an algorithm which positively answer the question by presenting graphs G_S and G_T. We then discuss the minimality of θ.

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Finding the p_3 Hull Number of a Chordal Graph

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Keywords: spread of disease, graph convexity, irreversible threshold process.

A set C of vertices of a graph G is p_3-*convex* if $v \in C$ for every path $uvw \in G$ with $u, w \in C$. The p_3 *interval* $I_3[v, w]$ of a pair of distinct vertices $v, w \in V(G)$ is the set formed by v, w and the vertices lying in some $v - w$ path of order three. For $S \subseteq V(G)$, let $I_3[S] = \bigcup_{v,w \in S}(I_3[v, w])$. If $S \subset V(G)$ and $I_3[S] = S_1, I_3[S_1] = S_2, ..., I_3[S_n] = V(G)$ then S is a p_3 *hull set* of G. The p_3 convex hull of S is the smallest p_3 convex subset of G containing S. The p_3 *hull number* of G is the minimum cardinality of a subset S whose p_3 convex hull is the entire graph.

The problem of finding the p_3 hull number is NP-complete for general graphs [1]. Using the irreversible conversion model [1] we present a solution for the p_3 hull number of chordal graphs.

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The Carathéodory number for the P_3 convexity

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Keywords: Carathéodory number, P_3 convexity.

Given a graph G and a collection $\mathcal{C}$ of subsets of $V(G)$, the pair $(G, \mathcal{C})$ is a *graph convexity* if $\emptyset, V(G) \in \mathcal{C}$, and $\mathcal{C}$ is closed under intersection. The sets of $\mathcal{C}$ are called *convex sets*. The *convex hull* of S, with respect to some convexity $\mathcal{C}$, is the smallest set $H_{\mathcal{C}}[S]$ in $\mathcal{C}$ containing S.

In a graph convexity $(G, \mathcal{C})$, the *Carathéodory number* is the smallest number c such that for every $S \subseteq V(G)$ and $p \in H_{\mathcal{C}}[S]$, there exists $F \subseteq S$ with $|F| \leq c$ such that $p \in H_{\mathcal{C}}[F]$. It is known that the Carathéodory number for the monophonic convexity is 1 for complete graphs, and 2 for other graphs, [2]. In [1], has been shown that $c = 2$ for the triangle path convexity. There are also results for a convexity in multipartite tournaments where $c \leq 3$, [3].

In this work, we show that the Carathéodory number for the P_3 (2-path) convexity is unbounded. Moreover, for every integer $k \geq 1$ we construct a graph having Carathéodory number k.

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Decompositions by Maxclique Separators

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Keywords: maximal clique separators, graph decompositions

Decompositions by clique separators are useful for efficiently solving graph problems in many classes of graphs. In (Decompositions by clique separators. *Discrete Mathematics*, 55, pages 221–232, 1985) R. Tarjan designed an $O(nm)$-time algorithm to decompose a graph with n vertices and m edges by clique separators. In a note at the end of his paper, he indicated how to modify the algorithm to perform decompositions by *maximal* clique separators.

We present a minimal example showing that the proposed modification is not sufficient. We also provide an algorithm retaining the same complexity.

Branch and Bound Algorithms for the Maximum Clique Problem

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Keywords: maximum clique, branch and bound

The Maximum Clique Problem (MC) is the problem of finding a clique of maximum size on a given graph. The problem is $\mathcal{NP}$-hard and there exists an $\varepsilon > 0$ such that the problem cannot be approximated in polynomial time up to a factor of n^{ε}, where n is the number of the vertices in the graph.

There are a number of proposed algorithms for the exact solution of MC whose performance is surprisingly good. Indeed, such algorithms are reported to effectively solve instances of practical interest in several domains. Branch and Bound schemes stand out in the literature as one of the best approaches for the exact solution of MC in practice.

More often than not, these algorithms are presented in the literature from an experimental standpoint, where running times for several testing benchmarks are given and commented upon, but little or none analytical results are given in support of the verified performance.

In this talk we review some Branch and Bound algorithms for the Maximum Clique Problem from a unifying point of view. This is part of an ongoing effort aiming at a better understanding of the performance of these algorithms, their strengths and limitations.

Graph Colorings and Scheduling Problems

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Keywords: weighted and bounded coloring, single machines, equal-time jobs.

We present some graph coloring problems that are generalizations of the classical vertex coloring, and can be applied to model several kinds of scheduling problems. So, in this work we review some variants of graph coloring, such as precoloring extension, μ-coloring, (γ, μ)-coloring, list coloring, list channel coloring, and so on, discussing their applications in scheduling theory. We also review some complexity results related to coloring and list channel coloring. A special scheduling problem, $1|p_j = p; r_j|-$ in 3-field notation, involving single machines and equal-time jobs is considered, and for this problem, we present two different models, the first one based on a collection of cliques and stable sets, and the second based on a variation of (γ, μ)-coloring and list channel coloring problems.

Matchings in Cartesian Product of Graphs and its Applications to Interconnection Networks

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Keywords: cartesian product, perfect matching, matching preclusion number.

The cartesian product of graphs is one of the operators in Graph Theory. In this work, we have developed several properties related to the existence of perfect and almost-perfect matchings in the product graph, by analyzing its factors. Properties of the matching preclusion number of the product graph were also studied. The matching preclusion number of a graph G, denoted by $mp(G)$, is the minimum number of edges that must be removed from $E(G)$ such that the resulting graph admits no perfect/almost-perfect matching. It is a topological criterion in interconnection networks, related to edge fault-tolerance; for instance, in some multiprocessor systems, where it is crucial that each processor has a special partner at any given time. The theoretical results obtained in this work lead to the determination of the matching preclusion number of several topologies based on the cartesian product: Hyper Petersen, Folded Petersen, Folded Petersen cube, Hyperstar, Star-cube and Hypercube (for this latter topology, it was already known). We have also shown that if two topologies with even number of vertices each are optimal with respect to the matching preclusion number then their cartesian product also has this property.

Extending the applicability of Rank-Divergence

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Keywords: Clique behavior, rank-divergence, iterated clique graphs

In the investigation reported in this abstract we extend the applicability of rank divergence (a stronger notion than clique divergence) by introducing some elementary operations that preserve rank divergence. The ideas involved in this study may be generalized and may lead to new, more powerful techniques to deal with such problems. We consider this work to be a step towards the solution of the projective plane conjecture that every Whitney triangulation of the projective plane is clique divergent.

For undefined terminology see ”Graph Relations, Clique Divergence and Surface Triangulations” Larrión et al. (2006). As usual, we denote by $N(x)$ the subgraph induced by the neighbors of x, and by C_n the graphs which is a cycle on n vertices. We say that $\mathbb{A}$ admits $\mathbb{B}$ if there is an admissible relation from $\mathbb{B}$ to $\mathbb{A}$. It is known that if $\mathbb{A}$ admits $\mathbb{B}$ and $\mathbb{B}$ is rank divergent, then so is $\mathbb{A}$. Then we have the following:

Theorem 1. Let xy be an edge of $\mathbb{A} = (A, \alpha)$, with $N(x) \cong C_4 \cong N(y)$ and assume α to be involutive ($\alpha^2 = 1$). Let $\mathbb{B}$ be the graph obtained from $\mathbb{A}$ by contracting the edges xy and $\alpha(x)\alpha(y)$. Then $K^2(\mathbb{A})$ admits $\mathbb{B}$.

Theorem 2. Let xy be an edge of $\mathbb{A} = (A, \alpha)$, with $N(x) \cong C_4$ and $N(y) \cong C_5$ and assume α to be involutive. Let $\mathbb{B}$ be the graph obtained from $\mathbb{A}$ by contracting the edges xy and $\alpha(x)\alpha(y)$. Then $K^4(\mathbb{A})$ admits $\mathbb{B}$.

Theorem 3. Let $a - b - y - c - d$ be an induced path in $\mathbb{B} = (B, \beta)$ such that $N(x) = \{a, b, y, c, d\}$ for some vertex x and assume β to be involutive. Let $\mathbb{A}$ be the graph obtained from $\mathbb{B}$ by replacing the edge xy by the edge bc and the edge $\beta(x)\beta(y)$ by the edge $\beta(b)\beta(c)$. Then $K^2(\mathbb{A})$ admits $\mathbb{B}$.

Examples of the usability of these theorems will also be presented.

The images of the clique operator and its square are different

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Keywords: clique graph, clique operator, octahedral graph.

Given a simple graph G, a set $C \subset V(G)$ is called a *complete* if all of its vertices are pairwise adjacent. And it is a *clique* if no other complete contains it. The *clique graph* of G has all the cliques of G as vertices and two of them are adjacent if and only if their intersection is not empty. If $\mathcal{G}$ denotes the class of all graphs, the function $K : \mathcal{G} \to \mathcal{G}$ that assigns to each graph its clique graph is called the *clique operator*. The exponential notation K^n will indicate the composition of the clique operator with itself n times.

It has been an open question for many years whether $K(\mathcal{G})=K^2(\mathcal{G})$. It has been suspected that the equality is false, and some partial results on that direction had been obtained [1].

Define the *n-dimensional octahedron* O_n as the complement of nK_2. It will be proved here, with the aid of [2], that O_4 is in $K(\mathcal{G})-K^2(\mathcal{G})$. The proof is broken into two steps. In first place, $K^{-1}(O_4)$ is characterized thanks to the fact that any graph there has O_3 as an induced subgraph. And then it is proved that no graph in $K^{-1}(O_4)$ is a clique graph, from which the claim that $O_4 \notin K^2(\mathcal{G})$ immediately follows.

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On weighted clique graphs

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Keywords: weighted clique graphs, graph classes structural characterization.

Let $K(G)$ be the clique graph of a graph G. A *m-weighting of* $K(G)$ consists on giving to each m-size subset of its vertices a weight equal to the size of the intersection of the m corresponding cliques of G. We will denote by $K_{m_1,\ldots,m_\ell}(G)$ the clique graph of G with weightings of sizes $m_1,\ldots,m_\ell$. In [1] and [2], 2-weighted clique graphs were considered in the context of chordal graphs. In this work we obtain a characterization of weighted clique graphs similar to Roberts and Spencer's characterization for clique graphs [3].

Some graph classes can be naturally defined in terms of their weighted clique graphs, for example clique-Helly graphs and their generalizations, and diamond-free graphs. The main contribution of this work is to characterize several graph classes by means of their weighted clique graph. We prove a characterization of hereditary clique-Helly graphs in terms of K_3 and show that $K_{1,2}$ is not sufficient to characterize neither hereditary clique-Helly graphs nor clique-Helly graphs. Similar results are obtained for split graphs and for chordal graphs and their subclasses UV graphs, interval graphs, indifference graphs, trees and block graphs.

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Parameterized Tractability of Cost Problems for X-of-Y Graphs.

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Keywords: X-of-Y Graphs, Parametrized Complexity, W[1]-hardness

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In this work, we show some polynomial cases for the minimum cost problem for X-of-Y graphs. Then, we prove that this problem is NP-hard. From the point of view of the parameterized complexity theory proposed by Downey and Fellows, we introduce two new problems: the k-maximum cost for X-of-Y graphs, and the k-cost for X-of-Y graphs, where we proved, unless $P = NP$, that the k-maximum cost problem for X-of-Y graphs is fixed-parameter intractable, and the k-cost problem for $X\text{-}of\text{-}Y$ graphs is fixed-parameter intractable even if the input is an X-of-Y tree.

Short covering codes and sum-free sets

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Keywords: covering code, extremal problem, sum-free sets, cyclic group.

Let $\mathbb{F}_q^3$ denote the set of all vectors $x = (x_1, x_2, x_3)$ with length 3 and components x_i taken on the field $\mathbb{F}_q$ with q elements, where q is a prime or a prime power. This set becomes a metric space by defining the *Hamming distance* $d(x, y)$ between the words x and y as the number of components in which x and y differ. A subset C in $\mathbb{F}_q^3$ is an *covering* of $\mathbb{F}_q^3$ iff for every vector x in $\mathbb{F}_q^3$, there is a vector y in C such that $d(x, y) \leq 1$. The number $K_q(3, 1)$ denotes the minimum cardinality of a covering of $\mathbb{F}_q^3$. Kalbfleish and Stanton, in 1969, showed that $K_q(3, 1) = \lceil q^2/2 \rceil$. The generalization for higher dimensions has been extensively investigated by many researchers since the paper by Taussky and Todd in 1948.

A closely related problem is described below. A subset H of $\mathbb{F}_q^3$ is a *short covering of* $\mathbb{F}_q^3$ when $\mathbb{F}_q.H = \{\alpha.h : \alpha \in \mathbb{F}_q \text{ and } h \in H\}$ is a covering of $\mathbb{F}_q^3$. The induced extremal problem $c_q(3, 1)$ is defined as the minimum cardinality of such subset H. Both problems can be reformulated in terms of graph theory.

In this talk, we introduce two extremal problems in combinatorial number theory aiming to discuss a known connection between the corresponding coverings and sum-free sets. Also, we provide several bounds on these maps which yield new classes of coverings, more precisely,

1. if q is odd, then $c_q(3, 1) \leq 6 \left\lceil \frac{q-1}{12} \right\rceil + 6 \left\lceil \log_4(\frac{q-1}{4}) \right\rceil + 3$;

2. if q is even, then $c_q(3, 1) \leq 6 \left\lceil \frac{q-1}{9} \right\rceil + 6 \left\lceil \log_4(\frac{q-1}{3}) \right\rceil + 3$.

Upper bounds and exact values on transposition distance in permutations

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Keywords: Genome rearrangement, Transposition distance, Lonely permutations

One of the main operations of genome rearrangement is the transposition (exchange of contiguous blocks). To date, no known polynomial algorithm to compute the minimum amount of operations needed to transform one sequence to another (transposition distance) between two permutations [1], or equivalently transform a sequence on the identity permutation. It is unclear whether this problem is NP-hard. The exact distance is known for few cases [2, 3, 4].

We show how to sort a lonely permutation type $U_{n,3}$ [3] applying $\lfloor \frac{n}{2} \rfloor + 1$ transpositions. Thus, if 4 divides $n+1$ then the transposition distance $d_t(U_{n,3}) = \lfloor \frac{n}{2} \rfloor + 1$, and if 4 does not divide $n+1$, we have that $\lfloor \frac{n}{2} \rfloor \leq d_t(U_{n,3}) \leq \lfloor \frac{n}{2} \rfloor + 1$. So, we obtained additional exact values for the transposition distance.

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Spectral properties of KK_n^j graphs

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Keywords: integral graph, characteristic polynomial of graph.

Let G be a simple graph. Let $A = A(G)$ and $D = D(G)$ be the adjacency matrix and the vertex degree diagonal matrix of G, respectively. Let $L = L(G) = D(G) - A(G)$ be the Laplacian matrix of G and $Q = Q(G) = D(G) + A(G)$ the signless Laplacian matrix of G. For each of the associated matrices of G, $M = A, L$, or Q, we call M-spectrum of G the spectrum of the matrix M. A graph G is called M-integral when all eigenvalues of M are integer numbers. In this work we present the KK_n^j graphs, obtained from two copies of the complete graph K_n by adding j edges, $1 \leq j \leq n$, between a vertex of one of the copies and j vertices of the other. We determine the M-characteristic polynomial of this graph. Finally, we give conditions for a graph KK_n^j to be M-integral.

An improvement on point location queries in a set of fat obstacles and on motion planning amidst fat obstacles

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Keywords: computational geometry, motion planning, sampling

Motion planning consists in tracing a motion for a robot that starts from a initial point q_{ini} and finishes at a destination point q_{end} in a static workspace, avoiding collision with the scenes' obstacles and finally returning the movement, if it exists. The motion planning problem has some variations, of which one of the most common is realistic input scenes.

There are mainly two philosophies that solve motion planning: explicit construction and sampling. The former consists in constructing the space explicitly and the latter consists in sampling points where the robot could be placed. In addition, the second philosophy has faster algorithms. Nevertheless, we don't have all the space sampled, thus we are dealing with partial information about the obstacles. This is a trade-off between the two main philosophies. We present in this work a faster way than Overmars et al. [1] to solve the variation of realistic input scenes where the objects are fat, i.e., have similar directional width in all directions, using the latter philosophy with advantages from both philosophies.

References

[1] A. F. van der Stappen, M. H. Overmars, "Motion planning amidst fat obstacles", in SCG'94: *Proceedings of the Tenth Annual Symposium on Computational Geometry*, New York, NY, USA: ACM, 1994, pp. 31–40.