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Welcome from Marina Groshaus (Chair)

Welcome to Buenos Aires for celebrating the fifth edition of the Latin American Workshop on Cliques in Graphs. The first Latin American Workshop on Cliques in Graphs was held in Rio de Janeiro in 2002, and was born in honor to Jayme Szwarcfiter in his sixty birthday. Ten years has passed since then, and this time we have the great pleasure to celebrate the 10th anniversary of the workshop along with Jayme's 70th birthday.

In its origins, the workshop was meant to foster interaction between the Latin-American graph theory and combinatorics researchers, whose interests include cliques, clique graphs, the behavior of cliques, and related issues. In this ten years the community has grown and the workshop has become not only a place for exchanging ideas on cliques, but a moment to strengthen the relationships between its community members as well as to start new relations.

We want to thank the Steering Committee for trusting us the organization of this edition of the workshop, the Program Committee for their help in organizing the sessions, the invited speakers for their enthusiastic participation, and all the authors for their contributed talks. We are also grateful to Rosiane de Freitas, Adriana Pimenta Figueiredo, Mitre Costa, Fábio Protti, and Celina Figueiredo for their collaboration in the organization of the social events.

Finally, we thank all the participants for their support to the conference, and we hope you have a pleasant, interesting and joyful stay. **Enjoy the workshop!**

Marina Groshaus (Chair)
On behalf of the Organizing Committee.

Cliques and Relatives – Bounds and Structures

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In this talk I will survey recent results concerning cliques in graphs and their relatives.

Solitaire Clobber played on cartesian product of graphs*

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Solitaire Clobber is a one-player combinatorial game where black and white stones are located on the vertices of a given graph. A move consists of picking a stone and clobbering another one of opposite color located on an adjacent vertex; the clobbered stone is removed from the graph and it is replaced by the picked one; the goal is to find a succession of moves that minimizes the number of remaining stones, when no move is possible.

A *configuration* Φ of a graph $G = (V, E)$ is a mapping $\Phi : V \rightarrow \{\bullet, \circ\}$ and we say that (G, Φ) is k-*reducible* if there exists a succession of moves that leaves at most k stones on the graph.

In 2008, Dorbec et al. [1] introduced a more restrictive question about Solitaire Clobber. A graph G is said to be *strongly* 1-*reducible* if: for any vertex v of G, for any configuration of G (provided $G \setminus v$ is non-monochromatic), for any color c (black or white) there exists a way to play that yields a single stone of color c on v. The strongly 1-reducible graph class is included in the class of graphs for which there exists a Hamiltonian path with ending point on v, for all $v \in V$.

These authors considered this problem with respect to the cartesian product of graphs where at least one of these graphs is a clique. The results are: all cliques of size $n \geq 3$ are strongly 1-reducible; if the graph is a multiple cartesian product of cliques (Hamming graph) then it is strongly 1-reducible, except hypercubes and cartesian product of K_2 with K_3; and if G is a strongly 1-reducible graph containing at least 4 vertices, then the cartesian product of G with any clique is strongly 1-reducible [1].

In this paper we show a generalization of the result of Dorbec at al. [1] by proving that if we have two strongly 1-reducible connected graphs G and H (both graphs with at least three vertices) then $G \Box H$ is strongly 1-reducible.

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Arboreal jump number of an order*

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Let $\mathcal{P} = (X, P)$ and $\mathcal{A} = (X, A)$ be finite orders, we call $\mathcal{A}$ an extension of $\mathcal{P}$ if $P \subset A$. If the Hasse diagram of $\mathcal{A}$ is a rooted tree, then $\mathcal{A}$ is an *arboreal extension* of $\mathcal{P}$. We say A has an *arboreal jump* at i if x_i and x_j are not comparable in P and x_i has a cover relation with x_j. The *arboreal jump number* of $\mathcal{P}$ is the least arboreal jump number among its arboreal extensions.

The jump number problem of a linear extension is very important in order theory and work scheduling. An early paper on the subject is that by Habib and Cogis [1]. Pulleyblank had shown that this problem is NP-complete for general orders [2]. Moreover, Mitas proved that it remains NP-complete for interval orders [3]. We define the arboreal jump number problem for partially ordered sets, which is a generalization of the jump number problem. The arboreal jump number problem has an order P as instance and the question is to determine the minimum number of arboreal jumps necessary to find an arboreal extension of P.

In this paper, we consider the arboreal jump number problem. Thus, we define the arboreal jump number, minimum and minimal arboreal extensions of an order $\mathcal{P} = (X, P)$, and describe several results for this problem, including $NP-$completeness for general orders. We describe an upper bound for the arboreal jump number for an order P in terms of the ground set X and the edges set of its Hasse diagram. We solve the problem for some classes, N-free orders, parallel orders and bipartite orders. We show that the maximal elements of a N-free order P are preserved in its minimum arboreal extensions. Finally, we also show that the arboreal jump number is not a comparability invariant.

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EPT GRAPHS ON BOUNDED DEGREE TREES

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An (h, s, t)-representation of a graph G consists of a collection of subtrees of a tree T, where each subtree corresponds to a vertex in G, such that (i) the maximum degree of T is at most h, (ii) every subtree has maximum degree at most s, (iii) there is an edge between two vertices in the graph G if and only if the corresponding subtrees have at least t vertex in common in T. The class of graphs that have an (h, s, t)-representation is denoted by $[h, s, t]$. It is known that $[\infty, 2, 2] = EPT$, $[3, 2, 2] = EPT \cap$ chordal and $[4, 2, 2] = EPT \cap$ weakly chordal.

It was an open question[1] to know the relationship between the maximum degree h of the host tree and the length n of the longest chordless cycle ($n \geq 4$). It is known that if $G \in [h, 2, 2]$, then G has no chordless cycle C_n for $n \geq h + 1$, since there is a unique EPT representation of C_n as a pie with n slices. The converse is true for $h = 3$, since G must be chordal. The converse is false for $h = 4$, since the graph $\bar{C}_6$ has no chordless cycle greater than 4 and $\bar{C}_6 \notin [4, 2, 2]$. It was an open question to know if the converse is true for $h \geq 5$. We prove that the converse is false for $h \geq 5$. That is, we find a family $F_{i,j,k}$, with $i, j, k \geq 1$, such that if $i + j + k + 2 = h$, then $F_{i,j,k} \in [h, 2, 2] - [h - 1, 2, 2]$ and $F_{i,j,k}$ is $\{C_n, n \geq h\}$-free.

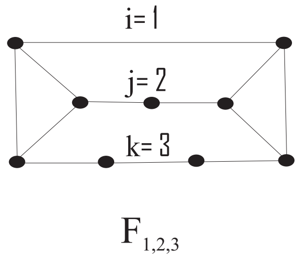

Figure 1: $F_{1,2,3} \in [8, 2, 2] - [7, 2, 2]$ and $F_{1,2,3}$ is $\{C_n, n \geq 8\}$-free.

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On basic chordal graphs and some of its subclasses

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Chordal graphs are defined as those graphs without induced cycles of length greater than or equal to four. Chordal graphs can also be characterized through clique trees. A *clique tree* of a graph G is a tree T such that its vertices are the cliques of G, and for every vertex v of G, the family $\mathcal{C}_v$ of cliques of G containing v induces a subtree of T. A graph is chordal if and only if it has a clique tree.

The image of the class of chordal graphs via the clique operator is another well known class, i.e., *dually chordal graphs*. This class can also be characterized in terms of trees. A *compatible tree* of a graph G is a spanning tree T such that every clique of G induces a subtree of T. A graph is dually chordal if and only if it has a compatible tree.

The clique operator not only relates chordal and dually chordal graphs, but also relates their characteristic trees. More precisely, every clique tree of a chordal graph is a compatible tree of its clique graph. However, it is not necessarily true that a compatible tree of the clique graph is a clique tree of the original graph.

A graph G is said to be *basic chordal* if it is chordal and its clique trees are exactly the compatible trees of $K(G)$. Two of the major results known about basic chordal graphs are a characterization of them that gives a method to efficiently decide whether a graph is basic chordal by looking at its minimal vertex separators, and the fact that the image through the clique operator of the class of basic chordal graphs is the class of dually chordal graphs [1].

Chordal graphs have subclasses such as DV and RDV graphs, that are characterized by the existence of special types of clique trees, namely, DV-*clique trees* and RDV-*clique trees*. The images of these classes through the clique operator, i.e., *dually* DV and *dually* RDV graphs, are in turn characterized by the existence of special types of compatible trees, namely, DV-*compatible trees* and RDV-*compatible trees*. In this context, *basic* DV and *basic* RDV graphs can be defined similarly to basic chordal graphs.

In this work, basic DV and basic RDV graphs are introduced and characterized, and their clique graphs are studied.

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Interval count of generalizations of threshold graphs

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A graph G is an *interval graph* if there is a correspondence between $V(G)$ and a family of intervals $\mathcal{R} = \{I_v \mid v \in V(G)\}$ of the real line such that, for all distinct $u, w \in V(G)$, $I_u \cap I_w \neq \emptyset \iff uw \in E(G)$. Such a family $\mathcal{R}$ is called an *interval model* of G. The class of interval graphs is well-known and its interest comes both from pure theoretical research and from its central role in several practical applications [3]. An order $(X, \prec)$ is an irreflexive and transitive binary relation $\prec$ on a set X. Interval orders are those orders defined by the transitive orientations of the complement of interval graphs, letting X be the vertex set and $x \prec y$ when vertex x is oriented towards vertex y.

The *interval count problem* is that of determining the smallest number of interval lengths required in an interval model of a given interval graph or interval order. Although there is intensive research on interval graphs, few results on the interval count problem are known. For instance, it is well-known that graphs with interval count one (unit interval graphs) can be recognized in linear-time using several different approaches, first of them dated from the sixties, whereas the complexity for the recognition of graphs with interval count two is still open. Moreover, the actual computation of the interval count has been determined only for certain classes, as trees, almost-$K_{1,3}$-free graphs (those free of induced $K_{1,3}$ except for the removal of one vertex), and generalizations of threshold graphs [2, 1].

In this work, we discuss why the class of split graphs consists of a natural candidate to have its interval count investigated and show that a particular subclass of it, another generalization of threshold graphs, has interval count also limited to two. We note that threshold dimension is two for graphs in this subclass and that there are interval graphs that are not split with threshold dimension two.

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Modelos DV que pueden enraizarse

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Un resultado clásico [1] establece que un grafo es *cordal* si y sólo si posee un árbol cuyos vértices son los cliques del grafo y tal que cada vértice del grafo induce un subárbol en el árbol. A los árboles antes mencionados les diremos *modelos*.

Dos subclases de grafos cordales son la clase de *grafos de intervalos* y la clase de *grafos DV* [2]. Un grafo es de *intervalos* si y sólo existe un modelo camino. Un grafo es *DV* si y sólo si existe un modelo DV es decir un árbol dirigido tal que la familia de subárboles inducidos por los vértices del grafo son caminos dirigidos.

Una clase de grafos intermedia entre las clases de grafos de intervalos y grafos DV es la clase de *grafos RDV* [2]. Un grafo es *RDV* si y sólo si existe un modelo DV que puede ser enraizado.

Es claro que todo modelo de intervalos puede ser enraizado en cualquiera de sus dos hojas o bajo ciertas condiciones en un vértice interno. Por otro lado, es fácil verificar que un modelo DV con 3 hojas puede ser enraizado. Es natural preguntarse cuando un modelo DV de un grafo con más de dos hojas puede ser enraizado o en caso de no ser enraizable si a partir de ese modelo es posible construir otro con al menos tres hojas que pueda enraizarse.

En este trabajo se presentan condiciones suficientes para grafos DV con leafage al lo sumo cuatro admitan modelos RDV con 3 o más hojas.

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Self-diclique digraphs

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Let $D = (V, A)$ be a digraph. Consider X and Y (not necessarily disjoint) nonempty subsets of vertices of D. We define a *disimplex* $K(X, Y)$ of D to be the subdigraph whose vertex set is $V(K(X, Y)) = X \cup Y$ and which an arc goes from every vertex of X to every vertex of Y (when $X \cap Y \neq \emptyset$, loops are not considered). A disimplex $K(X, Y)$ is called a *diclique* of D if $K(X, Y)$ is not a proper subdigraph of any other disimplex of D . The *diclique digraph* (or *diclique operator*) $\overrightarrow{k}(D)$ of a digraph D is the digraph whose vertex set is the set of all dicliques of D and $(K(X, Y), K(X', Y'))$ is an arc of $\overrightarrow{k}(D)$ if and only if $Y \cap X' \neq \emptyset$. We say that a digraph D is self-diclique if $\overrightarrow{k}(D)$ is isomorphic to D. These definitions were introduced by Erich Prisner in his book "*Graph Dynamics*" (Pitman Research Notes in Mathematics Series, 338. Longman, Harlow, UK, 1995), where the following open problem (number 39 on page 207) is posed:

Problem. *Are there, besides the directed cycles, more* $\overrightarrow{k}$*-periodic digraphs in the family of all finite strongly connected digraphs?*

In this talk, we

(i) exhibit an infinite family of self-diclique circulant digraphs for which one of its members is an Eulerian orientation of the graph of the regular octahedron. This family is a natural generalization of the example given by B. Zelinka (*On a problem of E. Prisner concerning the biclique operator.* Math. Bohem. 127 (2002), no. 3, 371–373),

(ii) briefly sketch a proof that this family is the only self-diclique in the set of all circulant digraphs and

(iii) show an infinite family of self-diclique non-circulant digraphs.

We conclude with some open problems on dicliques.

On Clique Graphs of Chordal Comparability Graphs

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The *clique graph* $K(G)$ of a graph G is the intersection of the maximal cliques of G. A well known characterization of clique graphs is that by Roberts and Spencer (1971). In addition there are characterizations for clique graphs of several graph classes. In this work, we add a new class to this list, by describing the clique graphs of chordal comparability graphs. It is based on a new characterization of chordal comparability graphs, in terms of their maximal cliques. We recall that clique graphs of chordal graphs have been already characterized, e.g. [1], [2], [3]. As for comparability graphs, only partial characterizations of subclasses, such as cographs [4], are known. The problem of characterizing the clique graphs of general comparability graphs remains open.

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The Clique Operator Considered as a Functor

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A *clique* of a graph is a maximal complete subgraph of the graph. The *clique operator* K transforms a graph G into the intersection graph of its cliques $K(G)$.

The clique operator K is not a functor in the category of graphs. It certainly maps the category of graphs into itself (if A is a graph, $K(A)$ is also a graph). It surely maps identity morphisms of graphs ($1_A : A \to A$) into identity morphisms of graphs ($K(1_A) = 1_{K(A)} : K(A) \to K(A)$). But in general, given a morphism of graphs $\alpha : A \to B$ there is no uniquely defined, induced morphism $K(\alpha)$ such that $K(\alpha) : K(A) \to K(B)$. This particular problem can be solved by enlarging the class of morphisms in the category: Allowing graph relations (multivalued homomorphisms of graphs) as morphisms in the enlarged category we can define a canonical morphism $K(\alpha) : K(A) \to K(B)$. However, even then, the composition rule for morphisms does not hold: Given $\alpha : A \to B$ and $\beta : B \to C$ it does not hold in general that $K(\beta \circ \alpha) = K(\beta) \circ K(\alpha)$.

Something else must be done. As it turns out we can define a *combinatorial homotopy* $f \simeq g$ among morphisms f and g of graphs namely: given $f : A \to B$ and $g : A \to B$ we say that f and g are homotopic whenever there is a path graph P_n on n vertices (for some n) and there is a morphism of graphs $H : P_n \boxtimes A \to B$ such that $H(1, a) = f(a)$ and $H(n, a) = g(a)$ for all $a \in A$.

This combinatorial homotopy is analogous to the notion of homotopy among continuous functions of topological spaces. This new notion of combinatorial homotopy on the one hand retains many of the properties of the topological homotopy and on the other hand, it is amenable from the combinatorial point of view. With this combinatorial homotopy, we can define the combinatorial homotopy equivalence of graphs and we can also take the quotient category of graphs by declaring two morphisms equal whenever they are homotopic. In this quotient category, the clique operator is finally a functor. Once K becomes a functor, we have all of the category theory at our disposal for studying clique-related problems.

This combinatorial homotopy can indeed be defined in a variety of natural ways all of them leading to the same underlying notion: it is, for example, the coarser equivalence relation that makes a graph and its pared graph (the one obtained by removing dominated vertices) isomorphic. In this talk we shall see several of these equivalent ways for defining our combinatorial homotopy and explore some of the consequences and challenges.

Clique graphs and topology

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Given a simple graph G, we associate to it a topological space $|G|$ as the geometric realization of a simplicial complex $\Delta(G)$, which has as simplices the complete subgraphs of G. We say then that two graphs G_1, G_2 are homotopic, and write $G_1 \simeq G_2$ whenever $|G_1| \simeq |G_2|$.

On the other hand, the clique graph $K(G)$ of G is the intersection graph of its maximal complete subgraphs. Finding conditions on G such that $G \simeq K(G)$ was a problem considered by E. Prisner in the 90's, and he gave two big classes of graphs with that property: dismantlable and clique-Helly. In this talk we survey the research about this problem, together with some results obtained by the author in joint work with F. Larrión and M. Pizaña.

A LOWER BOUND FOR THE BICLIQUE-CHROMATIC NUMBER*

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A *biclique* of a simple graph G is a maximal set of vertices that induces a complete bipartite subgraph of G with at least one edge. A *biclique-colouring* of G is a mapping that associates a colour to each vertex such that no biclique is monochromatic. If the mapping uses at most k colours we say that π is a *k-biclique-colouring*. The *biclique-chromatic number* of G is the least k for which G has a k-biclique-colouring. Biclique-colouring has a "hypergraph colouring version". A colouring of a hypergraph is a mapping that associates a colour to each vertex such that no hyperedge is monochromatic. Let $G = (V, E)$ be a graph and let $\mathcal{H}_B(G) = (V, \mathcal{E}_B)$ be the hypergraph whose hyperedges are $\mathcal{E}_B = \{K \subseteq V \mid K \text{ is a biclique of } G\}$ — $\mathcal{H}_B(G)$ is called the *biclique-hypergraph* of G. A biclique-colouring of G is a colouring of its biclique-hypergraph $\mathcal{H}_B(G)$.

Biclique-colouring is a difficult problem, being coNP-complete [1] even to check if a colouring of a graph is a biclique-colouring. In the present work, we define a graph invariant that is a lower bound for the biclique-chromatic number of a graph. A *blique* is defined as a set of vertices that induces a complete graph and whose closed neighborhood also induces a complete graph. The *blique number* of a graph is the size of its maximum blique. We prove the following several properties of bliques.

Lemma 1 *Let L be a blique of a graph G. Any pair of vertices of L is a biclique of G.*

Proposition 2 *The biclique-chromatic number of a graph is at least its blique number.*

Lemma 3 *The set of maximal bliques of a graph is a partition of its vertex set.*

Theorem 4 *The blique number of a graph can be determined in polynomial time.*

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The star and biclique coloring and choosability problems

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In this work we study the computational complexity of the star and biclique coloring and choosability problems, which are analogous to the clique coloring and choosability problems.

A *biclique* of a graph G is an induced complete bipartite graph with at least two vertices. A *star* of G is a biclique with a universal vertex. A *star (resp. biclique) k-coloring* of G is a k-coloring of G that contains no monochromatic maximal stars (resp. bicliques). Similarly, for a list assignment $L\colon V(G) \to \mathcal{P}(\mathbb{N})$ of G, a *star (biclique) L-coloring* is an L-coloring of G in which no maximal star (biclique) is monochromatic. If G admits a star (biclique) L-coloring for every list assignment L such that $|L(v)| = k$ $(v \in V(G))$, then G is said to be *star (biclique) k-choosable.*

We prove that the star k-coloring and k-choosability problems are Σ_2^p-complete and Π_3^p-complete for $k > 2$, respectively, even when their inputs are restricted to graphs with no induced C_4 or K_{k+2}. Every biclique of a C_4-free graph is a star, thus, as a corollary, we obtain that the biclique k-coloring and k-choosability problems on $\{C_4, K_{k+2}\}$-free graphs are also Σ_2^p-complete and Π_3^p-complete, respectively. Following, we study all these problems considering the inputs are restricted to some related classes, including: K_3-free graphs; P_3-free graphs; $\overline{P_3}$-free graphs; co-bipartite graphs; graphs with no induced 4-wheels, gems, or darts; split graphs; threshold graphs; and block graphs.

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The biclique graph of some classes of graphs

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In this work we study the biclique operator KB that maps each graph to its biclique graph. A *biclique* of a graph is a set of vertices inducing a maximal complete bipartite subgraph. For a graph G, the biclique graph $KB(G)$ of G is the intersection graph of the bicliques of G. That is, $KB(G)$ has a vertex for each biclique of G and two vertices of $KB(G)$ are adjacent whenever their corresponding bicliques intersect.

The biclique graph can also be thought as an operator between graphs. For a graph class $\mathcal{C}$, $KB(\mathcal{C})$ is the family of all the biclique graphs of the graphs of $\mathcal{C}$. Conversely, for a class of graphs $\mathcal{C}$, the family $KB^{-1}(\mathcal{C})$ is formed by those graphs G such that $KB(G) \in \mathcal{C}$.

In this work we study the biclique operator for different classes of graphs. The goal is, on one hand, to characterize $KB(\mathcal{C})$ and $KB^{-1}(\mathcal{C})$ and, on the other hand, to determine the computational complexity of the problem of deciding if a given graph belongs to $KB^{-1}(\mathcal{C})$ for some class $\mathcal{C}$.

In particular, we study subclasses of chordal graph. We prove that if G is a bipartite chordal graph, then $KB(G)$ is either chordal or their induced cycles with minimum length belong to a wheel. Also we analyze those graphs that belong to $KB^{-1}(\mathcal{C})$ for the class $\mathcal{C}$ of chordal graphs. Finally, we prove that it is coNP-complete to determine if $G \in KB^{-1}(\mathcal{K})$, where $\mathcal{K}$ is the class of complete graphs.

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INTEGER INDEX OF n-BROOM-LIKE GRAPHS.

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A broom $B(a;r)$ is a graph with $a+r$ vertices obtained attaching a vertices to the vertex v_1 of the path $P_r = v_1 \dots v_r$, where $a \geq 1$ and $r \geq 1$. We will consider v_r as the root of $B(a;r)$. A p-broom is a graph obtained identifying the roots of p brooms in a single vertex.

A generalization of these graphs is obtained by identifying each vertex of a clique of size $n \geq 3$ with the root of a broom $B(a_i;r_i)$, and it is called a n-broom-like. If $a_i = a$ and $r_i = r$, for all $1 \leq i \leq n$, the n-broom-like is the hierarchical product of K_n by $B(a;r)$, denoted by $K_n \sqcap B(a;r)$. Moreover, if $r=1$, the hierarchical product $K_n \sqcap B(a;r)$ coincides with the corona graph $K_n \circ \overline{K_a}$. In this case, n is the clique number and na is the coclique number.

We calculate the spectrum of $K_n \circ \overline{K_a}$ and characterize when it is an integral graph and when it is a non-integral graph with integer index. For any odd n we exhibit a $K_n \circ \overline{K_a}$ integral graph. This does not occur in the case where n is even, for example, all graphs in the family $\{K_4 \circ \overline{K_a}; a \geq 1\}$ are non-integral. We succeeded, however, to build an infinite family of integral corona graphs, $K_n \circ \overline{K_a}$, with n and a both even.

Furthermore, fixing the clique number $n \geq 3$ and a constant $c \geq 2$, we obtain a total order of the graphs $K_n \sqcap B(a;r)$, such that $a+r=c$, according to their indices.

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Dominating Induced Matchings

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Let G be a simple weighted undirected graph, i.e, a graph without loops and multiple edges with vertex set V, edge set E and a weight function $w : E(G) \to \mathcal{R}$. Given an edge $e \in E$, we say that e dominates itself and every edge sharing a vertex with e. An induced matching in G is a subset of edges such that each edge of G is dominated by at most one edge of the subset. The problem of determining whether a graph has a dominating induced matching, i.e., an induced matching that dominates every edge of the graph is also known in the literature as *dominating induced matching* (DIM for short) or *efficient edge domination*. This problem is NP-Complete. In this work we study the weighted version of DIM, this is, find a DIM M such that the sum of weights of its edges is minimum between all DIM's if any and we proposed an exact algorithm to determine a minimum DIM with time complexity $O(1.485^n m)$.

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Efficient Multiple Domination

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Given a graph $G = (V, E)$ and a set of vertices $D \subseteq V$, a vertice $v \in V$ is *dominated* by D if $|N[v] \cap D| \geq 1$. When $|N(v) \cap D| = 1$ for all $v \in V$, G is *efficiently dominable*. A generalization of this concept is called *efficient multiple domination*, which requires all vertices must be dominated by a set $D \subseteq V$ exactly k times. Some results on the efficient multiple domination are presented, including bounds for the size of efficient k-dominating sets, the complement and iterated line graphs of efficiently $(r + 1)$-dominable r-regular graphs and a $\mathcal{NP}$-completeness proof for the efficient multiple domination problem in arbitrary graphs.

Interval Digraphs

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The study of interval graphs is one of the most beautiful and popular parts of graph theory, with interesting applications, elegant characterization theorems, and ingenious recognition and optimization algorithms. Much of this still applies to variants such as proper or unit interval graphs. However, when it comes to digraphs, much of the appeal seems lost, with no forbidden structure characterizations, and no really efficient recognition algorithms. I will discuss new variants of interval digraphs and proper interval digraphs, which retain some of the elegance of interval graphs. In particular, I will describe a forbidden structure characterization of a class of interval digraphs, which also applies to classical interval graphs and provides a link between the theorems of Lekkerkerker-Boland and Fulkerson-Gross. I will also describe several open problems. These results are joint with Arash Rafiey, and some also with Tomas Feder and Jing Huang.

Well-covered complementary prisms graphs

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Let $G = (V, E)$ be a simple graph and $\overline{G}$ be the complement of G. The complementary prism of G denoted by $G\overline{G}$ is the graph formed from the disjoint union of G and $\overline{G}$ by adding the edges of a perfect matching between the corresponding vertices of G and $\overline{G}$. This class was introduced by Haynes et al. [4]. In [5] Plummer defines a graph to be *well-covered* if all its maximal independent sets have the same size. The *girth* of a graph G is the length of a shortest cycle of G. If G does not contain any cycles, its girth is defined to be infinite.

The complementary prisms is a class whose many parameters have not been investigated yet. Some studied parameters are vertex independence, distance and domination [1, 2, 3, 4]. We show that the well-covered complementary prisms, with the exception of $G \in \{K_1, K_2\}$, have girth < 5 and we presented sufficient condictions to construct well-covered complementary prism graphs. We prove other properties related to maximal independent sets in complementary prisms.

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Enumeración de conjuntos independientes maximales en grafos bisplit

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Un grafo *bisplit* es un grafo no dirigido *G(V,E)* cuyo conjunto de vértices (V) puede ser particionado en tres conjuntos, X, Y y Z, de manera tal que X, Y y Z son conjuntos independientes e $Y \cup Z$ induce un grafo *bipartito* completo. En este trabajo se utilizan las características propias de la clase de grafos *bisplit* para diseñar un algoritmo que permita resolver el problema de enumeración de conjuntos independientes maximales en un grafo *bisplit* en tiempo polinomial en función de la cantidad de conjuntos independientes maximales, problema conocido por pertenecer a la clase #P-Completo[1] para un grafo cualquiera.

Palabras clave: Grafo *bisplit*, Enumeración de conjuntos independientes maximales.

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On the k, i-coloring problem

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A *coloring* of a graph G is an assignment of colors (represented by natural numbers) to the vertices of G such that any two adjacent vertices are assigned different colors. The smallest number t such that G admits a coloring with t colors is called the *chromatic number* of G, and is denoted by $\chi(G)$. In a k, i*-coloring* of a graph we assign to each vertex a set of colors of size k instead of a single color, in such a way that the sets of two adjacent vertices intersect in i colors or less. The k, i*-chromatic number* of a graph G, noted $\chi_k^i(G)$, is the minimum number of colors needed for a k, i-coloring of G. This coloring was introduced by Mndez-Diaz and Zabala in 1999. This new coloring parameter generalizes the work of Hilton, Rado and Scott, who introduced the problem for $i = 0$ on planar graphs. In their seminal paper, Mndez-Diaz and Zabala studied upper bounds for $\chi_k^i(G)$ and extensions of properties of the classic coloring problem; further, they proposed a linear programming approach for the problem.
In this work, we relate the k, i-coloring of cliques to a still unsolved problem of coding theory (making a polynomial solution unlikely), give new upper and lower bounds for $\chi_k^i(G)$, and give a solution for cycles and cactus graphs for some values of k and i. Finally, we prove that given a graph G with treewidth bounded by a constant c and fixed k, i and j, it can be determined in linear time whether a k, i-coloring with j colors does exist.

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On $L(h, k)$-coloring b-core limited graphs

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An $L(h, k)$-*coloring* of a graph $G = (V, E)$ is an assignment f of non-negative integers to its vertices with the following restrictions: if $uv \in E$, then $|f(u) - f(v)| \geq h$ and; if $dist(u, v) = 2$, then $|f(u) - f(v)| \geq k$. The greatest integer used in an $L(h, k)$-coloring f of a graph is the *span* of f. The minimum span among all $L(h, k)$-colorings of a graph G is denoted by $\lambda_{h,k}(G)$. Griggs and Yeh (Labeling graphs with a condition at distance 2, *SIAM J. Disc. Math.* **5**, 1992, 586–595) introduced this special type of vertex coloring and conjectured that every graph with $\Delta \geq 2$ admits an $L(2, 1)$-coloring with $\lambda_{2,1} \leq \Delta^2$. Havet, Reed, and Sereni ($L(2, 1)$-labelling of graphs, *Proceedings of SODA'2008*, 621–630) proved this conjecture is true for all graphs with $\Delta \geq 10^{63}$.

A b-*core* is an induced subgraph of a graph where every vertex has degree at least b. One can easily verify whether a graph has a b-core by recursively removing vertices with degree less than b. A graph is b-*core limited* if it has no $(b + 1)$-core. Mulet et al. (Coloring Random Graphs, *Phys. Rev. Lett.* 89, 2002, 268701-4) used b-core limited graphs to solve the coloring problem for $G(n, p)$ graphs.

We show that $\lambda_{h,k}(G) \leq \Delta^2(k - 1) + \Delta(2kb + h - k - 1) - b^2 + b(3 - 2k)$ for a b-core limited graph G. Furthermore, we give two families of b-core limited graphs for which the Griggs and Yeh's Conjecture is true. The first one is formed by b-core limited graphs with $b = (1 - \epsilon)\Delta$, where $0 < \epsilon \leq 1$ is a constant, and $\Delta \geq \frac{1-\epsilon}{\epsilon^2}$. The second is the family of the b-core limited graphs with $b \leq \Delta - \sqrt{\Delta}$. Whereas the proof of Havet, Reed, and Sereni for the Griggs and Yeh's Conjecture only holds for graphs with **extremely** high values of Δ, for the families of graphs previously described the conjecture holds for small values of Δ (e.g., b-core limited graphs with $\Delta \geq 100$ and $b = 0.9\Delta$, or with $\Delta = 100$ and $b \leq \Delta - \sqrt{\Delta} = 90$).

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Total chromatic number of some families of graphs with maximum degree 3*

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A *k-total-coloring* of G is an assignment of k colors to the edges and vertices of G, so that adjacent or incident elements have different colors. The *total chromatic number* χ_T of G is the least k for which G has a k-total-coloring. Clearly, $\chi_T \geq \Delta+1$ and the well-known Total Coloring Conjecture states that $\chi_T \leq \Delta + 2$. The focus of this work are graphs with maximum degree 3.

Snarks are cyclically-4-edge-connected cubic graphs of chromatic index 4 which had their origin in the search of counterexamples to the Four Color Theorem. In this work, we define an infinite family of snarks with squares and show that all its members have total chromatic number 4. The construction of this family uses the following definition. *Bricks* are connected bridgeless graphs which have 4 vertices of degree 2 and all others of degree 3, that are subgraphs of cyclically-4-edge-connected cubic graphs. Let G_1 and G_2 be two bricks. A *junction* of G_1 and G_2 is any cubic graph obtained by adding a matching between the four vertices of degree 2 of G_1 and the four vertices of degree 2 of G_2. A snark can be obtained by a junction of two bricks when at least one brick has chromatic index 4. The dot product [1] of two snarks gives a way to obtain a brick with chromatic index 4, by deleting one special pair of edges of this product.

Moreover, we determine the total chromatic number of some families of graphs with maximum degree 3: Hexagonal-grid, Wall(5), Wall(6), Near-prism(1), and Near-prism(2) families. We point out that some of the members of the last two families have total chromatic number 5.

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Minimal 3×3 M-obstruction cographs

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Let M be a symmetric $m \times m$ matrix over $0, 1, *$. An M-partition of a graph G is a partition of the vertex set $V(G)$ into m parts $V_1, V_2, \ldots, V_m$ such that: (i) V_i is a clique (respectively independent set) if $M(i,i) = 1$ (respectively $M(i,i) = 0$); (ii) there are all possible edges (respectively non-edges) between parts V_i and V_j, $i \neq j$, if $M(i,j) = 1$ (respectively $M(i,j) = 0$); (iii) there are no restrictions between parts V_i and V_j, $i \neq j$, if $M(i,j) = *$. A graph G that does not admit an M -partition is called an M -obstruction. A minimal M-obstruction is a graph G which is an M-obstruction, but such that every proper induced subgraph of G admits an M-partition. In [1] it is has been shown that matrix partition problems for cographs admit polynomial time algorithms and forbidden induced subgraph characterizations. Also, the authors bound the size of a largest minimal M-obstruction cograph.

This work provides explicit characterizations of M-partitionable cographs, in terms of minimal obstructions, for all 3×3 matrices M.

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Geodetic Number Versus Hull Number in P_3-Convexity

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In this talk we study the graphs G for which the hull number $h(G)$ and the geodetic number $g(G)$ with respect to P_3-convexity coincide. These two parameters correspond to the minimum cardinality of a set U of vertices of G such that the simple expansion process that iteratively adds to U, all vertices outside of U that have two neighbors in U, produces the whole vertex set of G either eventually or after one iteration, respectively. We establish numerous structural properties of the graphs G with $h(G) = g(G)$, which allow the constructive characterization as well as the efficient recognition of all triangle-free such graphs. Furthermore, we characterize the graphs G that satisfy $h(H) = g(H)$ for every induced subgraph H of G in terms of forbidden induced subgraphs. (Talk based on work accepted at WG 2012.)

Elements of Geodetic Convexity Applied to Classes of Graphs and Some Results in Distance-Hereditary Graphs

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A family $\mathcal{C}$ of subsets of a finite set X is called a convexity on X if: (a) $\emptyset, X \in \mathcal{C}$; (b) closed under intersections. The elements of $\mathcal{C}$ are called convex sets. Given $A \subseteq X$, the convex hull of A is the smallest convex set of $\mathcal{C}$ containing A. The most natural graph convexities are path convexities, each defined by a set $\mathcal{P}$ of paths in G. The well-known minimum path convexity is also called geodetic convexity, where elements of $\mathcal{C}$ are called geodesically convex sets (also g-convex sets). Let G be a finite, simple, connected and undirected graph. Given vertices u and v in $V(G)$, the geodetic interval $I[u,v]$ consists of all vertices of $V(G)$ belonging to some minimum path between u and v. For $S \subseteq V(G)$, $I[S]$ is the union of all geodetic intervals $I[u,v]$ with $u, v \in S$. Define $I^k[S]$ as an operation on geodetic intervals as follows: $I^1[S] = I[S]$ and $I^k[S] = I[I^{k-1}[S]]$. For $S \subseteq V(G)$, the g-convex hull of S, denoted by $H(S)$, can be computed in the following way: $H(S) = I^k[S]$, where $I^k[S] = I^{k-1}[S]$. A subset $S \subseteq V(G)$ is a *Carathéodory set* of G if $H(S) \setminus \cup_{x \in S} H(S \setminus \{x\}) \neq \emptyset$. The *Carathéodory number* of G is defined as $c(G) = max\{|S| + 1 : S \text{ is a Carathéodory set of } G\}$.

Our goal is to study the classes of graphs defined in the following way:

(i) $\mathcal{I}_p^k = \{G : H(S) = I^k[S] \text{ for all } S \subseteq V(G) \text{ with } |S| \leq p\}$.

(ii) $\mathcal{H} = \{G : H(S) = \bigcup_{u,v \in S} H(\{u,v\}) \text{ for all } S \subseteq V(G)\}$.

In this work we show that $\mathcal{I}_n^1 = \mathcal{I}_2^1 \cap \mathcal{H}$ and $\mathcal{H} = \{G : c(G) \leq 2\}$.

A graph G is called distance-hereditary if distances in any connected induced subgraph of G are the same as they are in G. Let $\mathcal{DH}$ stand for the class of distance-hereditary graphs. We prove that $\mathcal{DH} \subseteq \mathcal{H}$. We also give a forbidden subgraph characterization of $\mathcal{I}_2^1$ restricted to distance-hereditary graphs. Finally, we show graphs in $(\mathcal{DH} \cap \mathcal{I}_n^k) \setminus \mathcal{I}_n^{k-1}$ for every integer k.

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The Radon number on graphs

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Given a graph G and a collection C of subsets of V(G), the pair (G, C) is a *graph convexity* if $\emptyset, V(G) \in C$, and C is closed under intersection. The sets of C are called *convex sets.* The *convex hull* of S, with respect to some convexity C, is the smallest set $H[S]$ in C containing S. Some different graph convexities have been considered in the literature. The most commom of then is the *geodetic convexity*, where convex sets, are closed under shortest paths.

We say that in a set S exists a *Radon partition* if we can write $S = S_1 \cup S_2$, with $S_1 \cap S_2 = \emptyset$, but $H[S_1] \cap H[S_2] \neq \emptyset$. In a graph convexity (G,C), the *Radon number* is the smallest number r such that every $S \subset V(G)$ with $|S| \geq r$ than S admits a Radon partition. A set R is called an *anti-Radon set* if it admits no Radon partition. It is known the Radon number for monophonic convexity [2] and triangle path convexity [1].

In this work, we will determine the Radon number for some graph classes in the geodetic convexity. In addition, we recognize anti-Radon sets of unit interval graphs in the same convexity. The recognition of anti-Radon sets for unit interval graphs is based on strategies developed at [3]. For the Radon number on P_3-convexity (for paths of lenght 2), we mention results of [4].

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Two Families of Cayley Graph Interconnection Networks*

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In this work, our interests are in the design and analysis of static networks. Static networks can be modeled using tools from Graph Theory. The graph is the interconnection network, the processors are the vertices and the communication links between processors are the edges connecting the vertices. There are several parameters of interest to specify a network: low degree, low diameter, and the distribution of the node disjoint paths between a pair of vertices in the graph. The degree relates to the port capacity of the processors and hence to the hardware cost of the network. The maximum communication delay between a pair of processors in a network is measured by the diameter of the graph. Thus, the diameter is a measure of the running cost.

Our goal is to propose two new families of Cayley graphs that can be used to design interconnection networks. The definition of Cayley graphs was introduced to explain the concept of abstract groups which are described by a generating set. The Cayley graphs are regular, may have logarithmic diameter, are maximally fault tolerant and have a rich variety of algebraic properties.

One such algebraic property is that Cayley graphs are vertex transitive, i.e., the graph looks the same when viewed from any vertex. One important consequence of the vertex transitivity is that a guest structure embedded in one region of the host network can be readily translated to another region without affecting the quality of the original embedding. Other algebraic properties are to be edge transitive or to be both vertex and edge transitive which we call a symmetric graph.

The family $H_{l,p}$ has been defined in the context of edge partitions, and subsequently shown to be composed by Hamiltonian Cayley graphs. We consider two families of Cayley graphs: $H_{l,p}$, and $H'_{l,p}$, a related family composed of sparser graphs. The p^{l-1} vertices of the graph $H_{l,p}$ are the l-tuples with values between 0 and $p-1$, such that the sum of the l values is congruent to 0 $mod\ p$, and there is an edge between two vertices having two corresponding pairs of entries whose values differ by one unit. In the sparser graph $H'_{l,p}$ one of such pairs is the last one. The graph $H'_{l,p}$ has diameter $D = (\lfloor \frac{p}{2} \rfloor (l-1))$ and we show an algorithm to calculate the diameter of graph $H_{l,p}$ of time $O(l)$. The established properties support the graphs $H_{l,p}$ and $H'_{l,p}$ to be good schemes of interconnection networks.

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A Pre-Processing Procedure for The Bicluster Graph Editing Problem

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The *Bicluster Graph Editing Problem* (BGEP) is a NP-complete problem such that, given a bipartite graph $G = (V, U, E)$ and an integer $k \geq 0$, asks whether it is possible to add and/or remove at most k edges in order to make G a union of complete bipartite subgraphs (bicliques). The concept of grouping data into biclusters arises in many contexts and different disciplines. Mathematical models based on the BGEP produce good solutions to problems in computational biology and contributes to the design of multicast networks.

Our contribution is to conduct a pre-processing procedure to fix some variables and generate new constraints to the problem. This is done based on the following theorem. Let a and b be vertices in a bipartite graph $G(V, U, E)$. If $d(a, b) \geq 4$, then there exists an optimal solution in which a and b belong to distinct bicliques.

Dijkstra's algorithm is used to calculate the distance between each pair of vertices. The pre-processing procedure can set decision variables or create cuts based on the distances according to the theorem. If vertices u and v are in different partitions, we can simply set the variable $x_{uv} = 1$. In the case where u and v are in the same partition U, the cuts $x_{uw} + x_{vw} \geq 1$, $\forall w \in V$ will be created.

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Clique-based and others IP models for graph coloring with constraints and scheduling

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In this work, we discuss about graph colorings with constraints on vertices and edges, NP-hard problems, so as to present integer linear programming formulations based on clique partition and other graph properties. We present some graph coloring problems that are generalizations of the classical vertex coloring, such as multicoloring, list-coloring, channel-assignment coloring, and others, discussing their correlations with scheduling theory and applications in wireless networks. The approach applying mathematical programming in the classical vertex coloring has received special attention in recent years, with several known IP formulations, such as formulations based on independent sets or cliques, partial orders, and using some properties as acyclic orientations or asymmetric representatives. Some computational strategies are presented also, as pre-processing, relaxations, cut and column generation, and approximate techniques.

ON THE CLIQUE OPERATOR COMPLEXITY *

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The *clique operator* K in a graph G defines the graph $K(G) = H$ such that H is the vertex intersection graph of the cliques (maximal complete sets) of G. The CLIQUE OPERATOR decision problem consists of an input with a graph H and the question whether there is a graph G such that $K(G) = H$. A graph G is *clique Helly* if the family of cliques of G satisfies the Helly property: for every pairwise intersecting subfamily the total intersection is nonempty. The family of cliques of a *clique-complete* graph has total intersection nonempty. Hamelink proved [2] that being clique Helly is a sufficient condition to be clique graph. Later, Roberts and Spencer gave [4] a necessary and sufficient condition for a graph $G = (V, E)$ to be a clique graph: the existence of a complete set cover for the edges set of G satisfying the Helly property. We notice that the clique family of a graph can be exponential. Szwarcfiter proved [5] that the recognition of clique Helly graphs is polynomial and together with Mello and Lucchesi [3] that the clique-complete graph recognition is NP-complete. The recognition of clique graphs has also been proved [1] to be NP-complete. In this talk we will discuss some complexity aspects of the clique operator. We will see some input constraints in which the problem remains NP-complete and some input constraints in which the problem is polynomial. We will see a large range where the clique operator complexity status is unknown. Some of these constraints involve the problem restricted to some classical classes of graphs, as for example the split graphs, in which the problem was recently established to be NP-complete. We will see some parts of the proofs of these results we consider important since they can enable us to establish NP-completeness for further classes. We will discuss our ideas for deciding if the recognition problem restricted to planar graphs class is in P or is NP-complete.

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ON THE P_3-LOCAL CARATHÉODORY NUMBER

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Let G be a finite, simple, and undirected graph and let S be a set of vertices of G. If every common neighbor of a pair vertices of S also lies in S, then S is P_3-*convex*. The P_3-*convex hull* $H_G(S)$ of S is the smallest P_3-convex set containing S. The P_3-*Carathéodory number* of G is the smallest integer c such that for every set S and every vertex u in $H_G(S)$, there is a set $F \subseteq S$ with $|F| \leq c$ and $u \in H_G(F)$.

Another invariant associated with the Carathéodory number is the local Carathéodory number. Let G be a graph and $S \subseteq V(G)$. The P_3-*local Carathéodory number* is the smallest integer l such that for every $u \in H_G(S)$ there is a set $F \subseteq S$ with $|F| \leq l$ and $u \in H_G(F)$.

In [1], it has been described a polynomial-time algorithm to determine the P_3-Carathéodory number of a tree. On the other hand, it has been proved that the problem becomes NP-complete for bipartite graphs. In [2], it has been proved that the decision problem corresponding to the local Carathéodory number is also NP-complete, for general graphs.

In this work, we study structural and algorithmic aspects of the P_3-local Carathéodory number for some subclasses and present a polynomial-time algorithm to determine the P_3-local Carathéodory number for trees.

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Uma Convexidade em Grafos Direcionados Acíclicos

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A convexidade é definida considerando-se uma coleção S de subconjuntos de $V(G)$ então o par (G, S) é uma convexidade em grafo se: $\emptyset \in S$, $V(G) \in S$, e S é fechado sob intersecção. Os conjuntos em S são chamados de conjuntos convexos. Na convexidade procuramos três parâmetros: número de convexidade, número convexo e número de envoltória.

A convexidade em grafos direcionados foi estudada na década de 70 especificamente em torneios [1, 2, 4]. Para torneios multipartidos temos um estudo da convexidade de caminhos de comprimento dois [3]. Prosseguimos com o estudo da convexidade em grafos direcionados definindo o que chamamos de convexidade de precessão. Esta é definida sobre os predecessores diretos de um vértice. Se $(uv) \in E(G)$, dizemos que u é um predecessor direto de v. Para tal convexidade determinamos intervalos para o número de convexidade e o número de envoltória dos grafos direcionados acícilicos e ainda mostramos que o número convexo para tais grafos é um problema NP-completo.

Ainda propomos o estudo da convexidade geodética a qual se relaciona com o menor caminho entre dois vértices. Como Chartrand, et al. caracterizam os grafos de ordem n e que possuem número de convexidade igual a $n-1$ gostaríamos de desenvolver um algoritmo de tempo polinomial para o número da envoltória geodética para os grafos direcionados acíclicos.

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AVD-TOTAL-COLORING OF COMPLETE EQUIPARTITE GRAPHS*

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Let $G := (V, E)$ be a simple graph. A *k-total-coloring* of G is a mapping $\phi : (V \cup E) \to \{1, 2, \ldots, k\}$ such that no adjacent vertices or adjacent edges receive the same color, and no incident elements receive the same color. Let $C(u) = \{\phi(u)\} \cup \{\phi(uv) : uv \in E(G)\}$ be the set of colors that *occurs* in a vertex u, $u \in V(G)$. If for any pair of adjacent vertices $u, v \in V(G)$, $C(u) \neq C(v)$, then ϕ is an *adjacent-vertex-distinguishing-total-coloring (AVD-total-coloring)*. The *adjacent-vertex-distinguishing-total-chromatic number (AVD-total-chromatic number)*, $\chi''_a(G)$, is the smallest number of colors for which a graph G admits an AVD-total-coloring.

The *AVD-total-coloring problem* consists of determining $\chi''_a(G)$ for a simple graph G. This problem was first introduced by Zhang et al. around 2005. The authors determined the AVD-total-chromatic number for some classic families of graphs, and conjectured that for a simple graph G, $\chi''_a(G) \leq \Delta(G) + 3$.

In this work, we consider complete equipartite graphs. A *complete equipartite graph*, $K_{r(n)}$, is a graph whose vertex set can be partitioned into r independent sets (parts) of cardinality n, such that any two vertices belonging to different parts are joined by an edge. We prove the following result:

Theorem 1. *Let $G := K_{r(n)}$ be a complete equipartite graph with $n \geq 2$ and $r \geq 2$. If G has even order, then $\chi''_a(G) = \Delta(G) + 2$; otherwise, $\chi''_a(G) \leq \Delta(G) + 3$.*

Initially, note that $\chi''_a(G) \geq \Delta(G) + 2$ since G has two adjacent vertices of maximum degree. In order to prove this theorem, we build a $(\Delta(G)+2)$-AVD-total-coloring for G of even order and a $(\Delta(G)+3)$-AVD-total-coloring for G of odd order. These colorings are obtained by decomposing G into a set of disjoint complete graphs and a set of bipartite graphs. In our proof, four cases are considered depending on the parity of n and r. In each case, we assign suitable edge-colorings to the set of bipartite graphs and an AVD-total-coloring to the set of disjoint complete graphs in such a way that the result is an AVD-total-coloring to G. Also, we prove that this coloring uses the required number of colors.

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Multicolored Ramsey Numbers in Multipartite Graphs

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A great challenge in graph theory has been the determination of the Ramsey numbers. Given positive integers n and m, recall that the celebrated Ramsey number $r(n, m)$ denotes the smallest natural number r such that every red-blue coloring of the edges of the clique K_r with size r yields either a red copy of K_n or a blue copy of K_m.

Many concepts and variants have been introduced in order to shed light on the computation of these numbers since 1940. In particular, Burger and Vuuren [Discrete Mathematics, 283 (2004), 37-43] introduced the following Ramsey-type problem. Let $K_{n\times m}$ denote the balanced, complete multipartite graph having n classes, each class with m vertices. Given positive integers j, n ,m, p, and q, the *set multipartite Ramsey number* $M_j(K_{n\times m}, K_{p\times q})$ is the smallest natural number c such that every red-blue coloring of the edges of $K_{c\times j}$ yields either a red $K_{n\times m}$ or a blue $K_{p\times q}$.

These numbers can be regarded as an extension of the classical Ramsey numbers. Indeed, note that $M_1(K_{n\times 1}, K_{m\times 1}) = r(K_n, K_m) = r(n, m)$, since $K_{n\times 1}$ is isomorphic to K_n. Several results arising from relationships with the numbers $r(n, m)$ are derived. Moreover, general bounds are obtained, in particular, including a general lower bound by using the probabilistic method.

In this work we extend the set multipartite to an arbitrary number of colors, as described below. The number $M_j(K_{n_1\times m_1}, K_{n_2\times m_2}, \ldots, K_{n_k\times m_k})$ denotes the smallest positive integer c such that for every coloring of the edges of $K_{c\times j}$ with k colors, there is always a monochromatic copy of $K_{n_i\times m_i}$ for any i, where $1 \leq i \leq k$.

We discuss the connections with the multicolored Ramsey numbers. Several results by Burger and Vuuren are extended, including general lower and upper bounds. Moreover, we also prove sharper upper bounds for certain class of parameters. In particular the bound $M_2(K_{2\times 3}, K_{2\times 3}) \leq 13$ is obtained by using a variant of the Turán numbers, improving $M_2(K_{2\times 3}, K_{2\times 3}) \leq 24$ by Burger and Vuuren.

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EDGE COLOURING BISPLIT GRAPHS

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A graph $G = (V, E)$ is *bisplit* graph if its vertex set can be partitioned into two sets $V = X + Y$ such that X is a stable set and Y induces a biclique (complete bipartite graph). Brandstädt et al. developed a polynomial time recognition algorithm [1]. Bisplit graph is a superclass of split graph and a subclass of comparability graph.

The *chromatic index*, $\chi'(G)$, of a graph G is the minimum number of colours needed to colour the edges of G such that adjacent edges have different colours. Let $\triangle$ be the maximum degree in G. Vizing [5] proves that $\chi'(G)$ is $\triangle$ or $\triangle + 1$. Graphs for which $\chi'(G) = \triangle$ are Class 1 and those for which $\chi'(G) = \triangle + 1$ are Class 2. This problem is NP-complete [3], and remains so for comparability graphs. Chen, Fu and Ko [2] show that split graphs of odd maximum degree are Class 1.

A graph G is *overfull* if $|E| > \triangle \lfloor \frac{|V|}{2} \rfloor$ where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. All overfull graphs are Class 2 [4]. G is *subgraph-overfull* if there is an overfull subgraph H of G with $\Delta(H) = \Delta(G)$. Obviously, subgraph-overfull graphs are Class 2. In this work, we show that a bisplit graph is Class 1 if it is not subgraph-overfull.

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On selective-perfectness of graphs

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Let $V' \subseteq V$. We denote by $G[V']$ the graph induced by V'. A *k-coloring* of G is a mapping $c : V \to \{1, \dots, k\}$ such that $c(u) \neq c(v)$ for all $uv \in E$. The smallest integer k such that G is k-colorable is called the *chromatic number* of G and is denoted by $\chi(G)$. Consider now a partition $\mathcal{V} = (V_1, V_2, \dots, V_p)$ of the vertex set V of G. We will denote by $(G, \mathcal{V})$ the graph G together with a partition $\mathcal{V}$ of its vertex set and call it a *clustered graph*. The sets $V_1, \dots, V_p$ are called *clusters* and $\mathcal{V}$ is called a *clustering*.

A *selective k-coloring* of G with respect to $\mathcal{V}$ is a mapping $c : V' \to \{1, \dots, k\}$, where $V' \subseteq V$ with $|V' \cap V_i| = 1$ for all $i \in \{1, \dots, p\}$, such that $c(u) \neq c(v)$ for all $uv \in E$. Thus determining a selective k-coloring with respect to $\mathcal{V}$ consists in finding a set $V' \subseteq V$ such that $|V' \cap V_i| = 1$ for all $i \in \{1, \dots, p\}$ and such that $G[V']$ admits a k-coloring. The smallest integer k for which a graph G admits a selective k-coloring with respect to $\mathcal{V}$ is called the *selective chromatic number* of G and is denoted by $\chi_{sel}(G, \mathcal{V})$. It is obvious to see that $\chi_{sel}(G, \mathcal{V}) \leq \chi(G)$ for every clustering $\mathcal{V}$ of V. The selective coloring problem it is known to be NP-hard even in the disjoint union of paths of length three [1].

We define in this work the notion of selective perfectness and strong selective perfectness, and characterize (strong) selective perfect graphs by matrix properties and by forbidden configurations. We also study the recognition problem for each class, and the complexity of the selective coloring problem on selective perfect and strong selective perfect graphs.

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Feedback Vertex Set is NP-complete for Reducible Flow Hypergraphs

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A directed hypergraph $H = (V, A)$ is a pair [1, 2], such that V is a non-empty finite set of vertices and A is a set of hyperarcs, where a hyperarc $a = (X, Y)$ is an ordered pair with X, Y non-empty subsets of V. These structures can be used [2, 3] to model parallel processes with precedence restrictions.

The FEEDBACK VERTEX SET PROBLEM for hypergraphs has as input a directed hypergraph $H = (V, A)$ and a positive integer k, and the question is whether there is a set $S \subset V$ such that the removal of S from H produces an acyclic hypergraph.

In this work we are concern with FEEDBACK VERTEX SET PROBLEM for flow hypergraphs, a subclass of directed hypergraphs defined in [2]. We prove that FEEDBACK VERTEX SET PROBLEM is NP-complete even for reducible flow hypergraphs.

The proof is a reduction from the 3–satisfiability problem with at most 3 occurrences per variable, which is known to be a NP-complete problem [4].

We also exhibit a polynomial-time $\frac{1}{m}$-approximation for finding a minimum feedback vertex set of a flow hypergraph, where m is the maximum number of hyperarcs which a vertex of H belongs to.

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Complexidade de Problemas de Conexão de Terminais

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Neste trabalho apresentamos algumas versões do problema de conexão de terminais. Esse problema está intrinsecamente relacionado ao problema da árvore de Steiner em grafos, o qual consiste em encontrar uma árvore de tamanho mínimo que conecta um subconjunto de vértices W de G. A versão de decisão do problema da árvore de Steiner pertence à classe NP-completo. No entanto, quando o tamanho de W é limitado por uma constante k, o algoritmo de Dreyfus& Wagner computa uma árvore de Steiner para W em tempo $O(n^3 + n^2 2^{k-1} + n3^{k-1})$.

Dado uma árvore T, definimos alguns vértices especiais em T:

- se $d(v) = 2$ então v é denominado *elo*;
- se $d(v) > 2$ então v é denominado *roteador*.

A partir dessas definições, formulamos o seguinte problema:

Problema: Conexão de Terminais em Grafos (CTG)
Dados um grafo conexo G, um subconjunto de vértices W de G e dois inteiros l e r, existe algum subgrafo conexo e acíclico de G que contenha W com no máximo l vértices elos e r vértices roteadores?

As motivações do estudo desse problema são diversas assim como os outros problemas de conectividade, tais como comunicações em multicast, projeto de circuitos VLSI, na biologia computacional nos estudos de aproximação genética etc. Em particular, um exemplo hipotético referente ao problema CTG é a questão de segurança de tráfico de informação entre computadores numa rede, onde quanto menor o número de roteadores menor é a chance de uma mensagem ser interceptada na rede por intrusos mal-intencionados. Portanto, acreditamos que o problema CTG definido é fascinante em suas aplicações no contexto atual.

Dentre os resultados obtidos mostramos que: (a) CTG permanece NP-completo mesmo quando fixado um valor constante para o parâmetro l; (b) CTG permanece NP-completo mesmo quando fixado um valor constante para o parâmetro r; (c) CTG é polinomial para o caso onde r e l são fixos.

Searching for a NP-Complete Probe Graph Problem*

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A *probe graph* for a graph class $\mathcal{C}$ is a graph $G = (V, E)$, such that there exists an independent set $N \subseteq V$ into which it is possible to add some edges between vertices of N in order to obtain a graph G' belonging to $\mathcal{C}$[2]. If the independent set N is given as input, we have a special case of *graph sandwich problem*[1]. Our interest rests in this version of probe graph recognition problem.

A *graph partition* of a graph $G = (V, E)$ is a partition of $V(G)$ into a number of parts. A *graph partition problem* consists in finding a graph partition where the parts satisfy some internal or external contraints. A *three nonempty part problem* is a graph partition problem, such that $V(G)$ must be partitioned in exactly three nonempty parts. All possible such nonempty part problems, reflecting the various combinations of internal and external contraints, are classified as polynomial or NP-complete in both recognition and sandwich versions [3].

We focused on probe three nonempty part recognition problems, for which its sandwich version is NP-complete and its recognition version is polynomial. We show that most of those probe problems have a behavior strongly similar to their recognition version, despite being a special case of a NP-complete sandwich problem. Finnaly, we compare this polynomial behavior to some yet unclassified problems, for instance, *probe clique cutset*, identifying the dissimilarities and setting these as candidate to be NP-complete.

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(k, ℓ)-SANDWICH PROBLEM: WHY NOT ASK FOR SPECIAL KINDS OF BREAD? *

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In 1995, Golumbic *et al.* formulated the famous SANDWICH PROBLEM as follows: Given two graphs $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ and a property Π is there a graph $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$ and so that G satisfies the property Π? The graph G is called *sandwich graph.* Note that in these problems it is chosen a special filling for the sandwich.

The questioning we make here is: why not choose a particular kind of bread for a special filling of our sandwich? So, we will present a new work proposal related to graph sandwich problems: a generalized version that we called *graph sandwich problems with boundary conditions.* Our goal is to determine the complexity of the SANDWICH PROBLEM when beforehand we know that G^1 satisfies a property Π^1 and G^2 satisfies a property Π^2, where Π^1 and Π^2 are called *boundary conditions.* Thus, we define the SANDWICH PROBLEM FOR PROPERTY Π WITH BOUNDARY CONDITIONS Π_1 AND Π_2, denoted by (Π^1, Π, Π^2)-SP, as the sandwich problem in which the input consists of two graphs: $G^1 = (V, E^1)$ satisfying property Π_1 and $G^2 = (V, E^2)$ satisfying property Π_2. Hence, $(*, \Pi, *)$-SP denotes the general SANDWICH PROBLEM FOR PROPERTY Π, where the notation $*$ means that graphs $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ do not satisfy necessarily any specified property.

In this paper we work with a particular property Π: "to be a (k, ℓ)-graph". A graph is (k, ℓ) if its vertex set can be partitioned into at most k independent sets and at most ℓ cliques. Brandstädt *et al.* proved that the RECOGNITION PROBLEM FOR (k, ℓ)-GRAPHS is NP-complete for $k \geq 3$ or $\ell \geq 3$ and polynomial time solvable otherwise. When G is chordal, Hell *et al.* proved that the RECOGNITION PROBLEM FOR CHORDAL-(k, ℓ) GRAPHS can be done in polynomial time. Furthermore, (Dantas *et al.*) proved that the SANDWICH PROBLEM FOR (k, ℓ)-GRAPHS is NP-complete for $k \geq 2$ and $\ell \geq 1$.

However, using this new proposal we have good news proving that the (CHORDAL, (k, ℓ), CHORDAL)-SP is polynomial time solvable for any k, ℓ fixed. Moreover, we can generalize this result by requiring G^1 to be a *perfect graph* and G^2 to have a polynomial number of cliques.

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