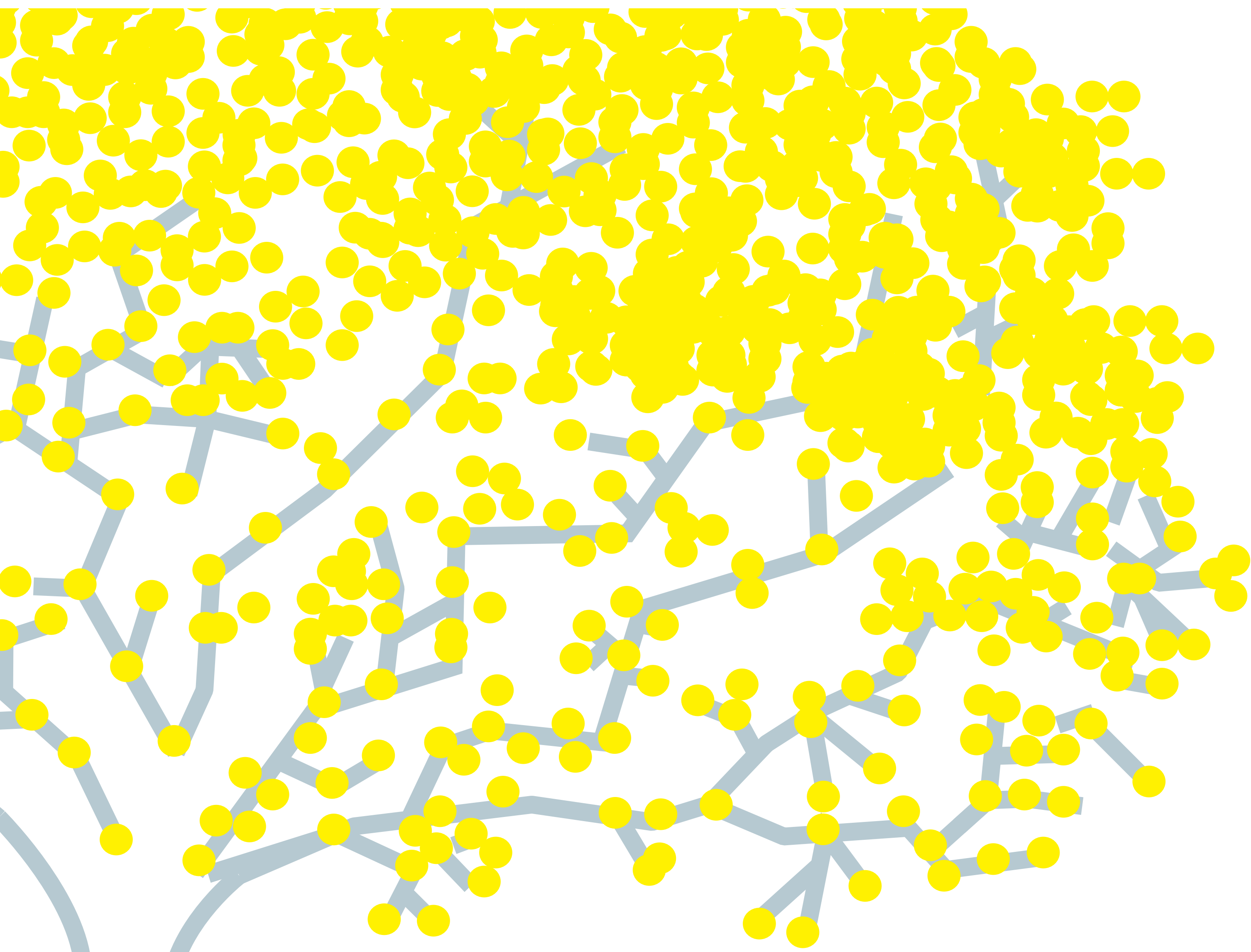

ABSTRACTS

Latin American Workshop on Cliques in Graphs, 2014

November 9-12

Pirenópolis-GO, Brazil

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VI Latin American Workshop on Cliques in Graphs
November 9-12, 2014
Pirenópolis, Goiás, Brazil.

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Preface

These abstracts were selected for presentation at LAWCG14, that will be held in Pirenópolis-GO, Brazil. The Latin American Workshop on Cliques in Graphs (LAWCG) series was started in Rio de Janeiro/Brazil (2002). The next editions were La Plata/Argentina (2006), Guanajuato/México (2008), Itaipava/Brazil (2010) and Buenos Aires/Argentina (2012).

The workshop is meant to foster interaction between the Latin-American graph theory and combinatorics community, whose research interests include cliques, cliques graphs, and related issues. In this sixth edition, the workshop has approximately 75 participants and 48 contributed talks.

Following an established tradition, a special issue of Matemática Contemporanea will be devoted to a selection of refereed full papers from the event. The deadline for submission to this issue will be posted in due time on the LAWCG14 website http//www.inf.ufg.br/lawcg14.

We would like to thank all members of Program and Organizing Committees. Especially, we want to thank the Steering Committee for entrusting us the organization of this edition of the workshop.

We are grateful to all participants for their contributions and particularly to the invited speakers. We would also like to thank our sponsoring agencies. The financial support received from CNPq, Capes and FAPEG was essential to this event.

Last but not least, we sincerely thank Instituto de Informática of Universidade Federal de Goiás and Instituto Federal Goiano of Rio Verde-GO by the received support.

Finally, we wish all participants have an interesting and stimulating event and a joyful stay at Pirenópolis.

Márcia Cappele and Erika Coelho (Conference Chairs)

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Cliques in Parameterized Hardness

Michael R. Fellows

Charles Darwin University, Australia

Problems about cliques underpin most hardness results in parameterized complexity, both in the sense of W-hardness, and in the "more modern" optimality program. The talk will survey the basic ideas, and give a how-to tutorial on proving $W[1]$-hardness results by reductions from Multicolor Clique.

Partitioning Distance Hereditary Graphs into stable sets and cliques

João Thompson[1] Loana Nogueira[1] Fábio Protti[1]
Raquel Bravo[2]

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In this work we consider the problem of partitioning a graph into k independent sets and l cliques, known as the (k, l)-*Partition Problem*, which was introduced by Brandstädt, and generalized by Feder, Hell, Klein and Motwani as the M-Partition Problem. Brandstädt proved that, given a graph G, it is NP-Complete to decide if G is a (k, l)-graph for $k \geq 3$ or $l \geq 3$. Particularly, we consider a subclass of perfect graphs: *Distance Hereditary Graphs* (DHG), which consists of graphs with isometric distances (every induced path between two vertices has the same length). We present a characterization of $(k, 1)$-DHG in terms of forbidden subgraphs, i.e., minimal obstructions.

For the sake of characterizing $(k, 1)$-DHG, we make use of the characterization of (k, l)-cograph and restricted our search to those graphs that are DHG and are not cographs - denoted as *Special Distance Hereditary Graphs* (SDHG). Therefore our goal is to prove the following theorem:

Theorem 1 *Let G be a SDHG graph. Then G admits a $(k, 1)$-partition iff G is (G_1, G_2)-free.*

Here, G_1 and G_2 represent two special infinite graph families.

Disimplicial arcs, transitive vertices, and disimplicial eliminations

Martiniano Eguía[1*] **Francisco J. Soulignac**[2†]

[1] DC–FCEN, Universidad de Buenos Aires
[2] CONICET and Universidad Nacional de Quilmes

In this talk we consider the problems of finding the disimplicial arcs of a sparse digraph and recognizing some interesting graph classes defined by their existence. A *diclique* of a digraph G is a pair $V \to W$ of sets of vertices such that $v \to w$ is an arc for every $v \in V$ and $w \in W$. An arc $v \to w$ is *disimplicial* when $N^-(w) \to N^+(v)$ is a diclique. For $E \subseteq E(G)$, a sequence $S = v_1 \to w_1, \ldots, v_k \to w_k \subset E$ is a *disimplicial E-elimination scheme (E-DES)* when $v_i \to w_i$ is disimplicial in $G_k = G \setminus \{v_1, w_1, \ldots, v_k, w_k\}$. If no edge of E is disimplicial in G_k, then S is *maximal*, while S is *perfect* when G_k is empty.

In the first part we show that the problem of finding the disimplicial arcs is equivalent, in terms of time and space complexity, to that of locating the transitive vertices. As a result, an $O(\alpha m)$ time and $O(m)$ space algorithm to find the bisimplicial edges of bipartite graphs is obtained, where m and α are the number of edges and the arboricity of the input graph, respectively. This improves upon the previous $O(nm)$ time and $O(m)$ space algorithm for sparse graphs (M. Bomhoff and B. Manthey. Bisimplicial edges in bipartite graphs. *Discrete Appl. Math.*, 161(12):16991706, 2013.)

In the second part, we develop two simple algorithms to build disimplicial elimination schemes. The first algorithm finds a maximal $E(G)$-DES in $O(\min\{\Delta\eta, m\}m)$ time, while the second one finds a maximal E-DES in $O(\alpha m)$ for any given matching E. Here Δ is the maximum among the degrees of the vertices and $\eta \leq m^{1/2}$ is the h-index of the graph. Both algorithms can be used to solve the respective problems of finding perfect eliminations schemes of bipartite graphs. The previous best algorithms for this problem on sparse graphs run in $O(m^2)$ time and $O(nm)$ time, respectively. (M. Bomhoff. Recognizing sparse perfect elimination bipartite graphs. *Lecture Notes in Comput. Sci.* 6651:443–455, 2011.)

Finally, we study two classes related to perfect disimplicial elimination digraphs, namely weakly diclique irreducible digraphs and diclique irreducible digraphs. A digraph G is *weakly diclique irreducible* when every arc of G belongs to a diclique that contains a disimplicial arc, while it is *diclique irreducible* digraphs as those digraphs in which every maximal diclique has a disimplicial arc. We show that the former class is related to finite posets, while the latter corresponds to dedekind complete finite posets.

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Experimental Analysis of Exact Algorithms for the Maximum Clique Problem

Cleverson Sebastião dos Anjos* Alexandre Prusch Züge†
Renato Carmo‡

Universidade Federal do Paraná

There are a number of proposed algorithms for the exact solution of the Maximum Clique (MC) problem which are reported to effectively solve instances of practical interest (some of them of considerable size) in several domains. Among them, branch and bound based schemes stand out as the best approach in practice.

One problem in comparing these algorithms is the way their merits are presented in the literature. Generally speaking, each author presents the outcome of their work by providing the results obtained by carrying out experiments concerning "their algorithm". The resulting comparison of experimental data, then, compares data from different implementations running under different computational environments.

Eight of these branch and bound algorithms for MC were described in [1] under a unifying conceptual framework which leads naturally to an unified implementation of them as parameterized versions of a general branch and bound routine.

Although this implementation is particularly well suited for a direct confrontation of the performances of the algorithms such was not the main intent of their work. As the authors themselves put, the intent of the experimental results was only to give to the reader an understanding and sense of dimension of the practical differences between the different approaches.

In this work we apply the concepts of Experimental Algorithm Analysis as exposed in [2] in order to perform a more thorough experimental structured confrontation of the algorithms studied in [1] by, but not limited to, defining an experimental process, establishing a structured testing environment, applying an experimental design model in a systematical manner and displaying the data gathered under different perspectives. While performing a comparison between the algorithms from [1] we also aim at reporting our "hands on" experience with some of the ideas and methodology from [2].

References

[1] Renato Carmo and Alexandre Züge. Branch and bound algorithms for the maximum clique problem under a unified framework. *Journal of the Brazilian Computer Society*, 18(2):137–151, December 2012.

[2] Catherine C. McGeoch. *A guide to experimental algorithmics*. Cambridge University Press, 2012.

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Reversible Processes on Graphs*

Mitre C. Dourado[1,3] **Carlos V.G.C. Lima**[2,3]
Jayme L. Szwarcfiter[1,2,3]

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Given a finite, simple, undirected and connected graph G and a function $f : V(G) \to \mathbb{N}$ we study the iterative process on G such that, given an initial vertex labelling $c_0 : V(G) \to \{0, 1\}$, each vertex v changes its label if and only if at least $f(v)$ neighbors have the different label. The transitions occur in synchronous way for each integer time step $t \geq 0$. Such processes model opinion dissemination and have been studied under names such as local majority processes or iterative polling processes in a large variety of contexts, especially in distributed computing.

It is known that these processes reach periodic behavior after a polynomial number of time steps, called transient length. In this work, we give a tight upper bound for the transient length of such processes.

We also study the problem of to find the minimum number $r_f(G)$ of vertices with initial state equals 1, such that, during the process on G, every vertices reach state 1. Given a constant k, we show that is NP-complete to determine if $r_f(G) \leq k$ even if G is a bipartite graph with $\Delta(G) \leq 3$ and $f : V(G) \to \{1, 2, 3\}$. We also prove that is NP-complete to determine if $r_f(G) \leq k$ even if G is a cubic planar graph and $f : V(G) \to \{3\}$. The last result comes from relationship between $r_f(G)$ and the size of a minimum covering of G, $\beta(G)$, where we show that $r_f(G) = \beta(G)$, if there is a minimum covering that does not induce an independent set in G, or $r_f(G) = \beta(G) + 1$, otherwise.

Keywords: reversible processes, transient length, R_f-Conversion Set Problem.

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On the complexity of the Cluster Deletion problem for several graph classes*

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A *cluster graph* is a graph in which every connected component is a clique. The *cluster deletion problem* (CDP) (resp. *weighted cluster deletion problem* (WCDP)) asks for the minimum number (resp. weight) of edges that can be removed from an input graph to obtain a cluster graph. We summarize in the following table some known results and the results obtained in this work, that are boldfaced in the table.

Class	CDP	Reference	WCDP
General	NP-c	Shamir, Sharan, and Tsur, 2004	NP-c
Complete split	**P**		**NP-c**
3-split	**P**		**NP-c**
Split	**P**		**NP-c**
P_5-free chordal	**NP-c**		**NP-c**
Block	**P**		**P**
Interval	?		**NP-c**
Proper interval	**P**		?
Paths of cliques	**P**		**P**
Trees of cliques	**P**		**NP-c**
Cographs	P	Gao, Hare, and Nastos, 2013	**NP-c**
P_4-reducible	**P**		**NP-c**
$\Delta = 3$	P	Komusiewicz and Uhlmann, 2012	?
C_4-free with $\Delta = 4$	NP-c	Komusiewicz and Uhlmann, 2012	NP-c
(C_5, P_5)-free	NP-c	Gao, Hare, and Nastos, 2013	NP-c
$(2K_2, 3K_1)$-free	NP-c	Gao, Hare, and Nastos, 2013	NP-c
$(C_5, P_5, \text{bull}, \text{4-pan}, \text{fork}, \text{co-gem}, \text{co-4-pan})$-free	NP-c	Gao, Hare, and Nastos, 2013	NP-c

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†Currently in "Délégation" at the INRIA Nancy - Grand Est 2013-2015.

Approximative algorithms for the maxcut of chordal graphs

Luerbio Faria*[1] **Rubens Sucupira**†[1] Sulamita Klein‡[23]

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[2] Instituto de Matemtica/UFRJ
[3] COPPE/Sistemas/UFRJ

Given a graph, the SIMPLE MAX-CUT problem asks to find a partition of its vertex set into two disjoint sets, such that the number of edges having one endpoint in each set is as large as possible. It is known that the SIMPLE MAX-CUT decision problem is NP-complete for general graphs and there is a polynomial time (1/2)-approximation algorithm to solve this problem. In particular, [1] proved that this problem remains NP-complete for split graphs. A split graph is a graph whose vertex set admits a partition into a stable set and a clique. [2] developed a semidefinite programming approximation algorithm with approximation ratio of 0,87856 to solve the SIMPLE MAX-CUT problem for general graphs. In this paper we show a polynomial time (2/3)-approximation algorithm for simple maxcut of split graphs and deterministic algorithms for simple maxcut of full (k,n)-split graphs using only simple combinatorial methods. Furthermore, we use the perfect elimination ordering of a chordal graph G to find an approximation algorithm to solve the SIMPLE MAX-CUT problem on chordal graphs with ratio $1 - \frac{D}{m}$, where m is the number of edges of G and D is the number of edges on the maximum clique of G that don't are in the maximum cut of that clique.

References

[1] BODLAENDER, H. L. AND JANSEN, K. On the complexity of the Maximum Cut problem, Lecture Notes on Computer Science, Volume 775 (1999) 769-780.

[2] GOEMANS, M. X.; WILLIAMSON, D. P. Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming. Journal of the ACM (JACM), v. 42, n. 6, p. 1115-1145, 1995.

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Distance coloring problems, spatial properties and feasibility conditions

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Graph coloring composes a large and important class of combinatorial optimization problems that has been extensively studied in the literature. One of its key applications is in the planning of resource allocation in mobile wireless networks. In this work, we present some theoretical graph coloring models, where the coloring should respect certain geographic and technological distance constraints. We show these coloring problems with distance constraints from the geometric distance point of view, that is, as the positioning of the vertices on the nonnegative integer line (or, as the immersion of the graph in 1-dimension), where the point on the line corresponds to the color to be assigned to a vertex, according to the distance between adjacent vertices. In addition, we have demonstrated for some classes of graphs, when these problems have, or do not have, feasible solutions.

Choosability for restricted list coloring problems

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Coloring problems with special constraints, adds a set of additional conditions on how the colors should be assigned to the vertices, edges, or both. Among these problems, the list coloring problem is a proper coloring of a graph $G = (V, E)$ in which to each vertex $v \in V(G)$ is associated a list of allowed colors $L(v)$. A list coloring is a choice function that maps every vertex v to a color in the list $L(v)$. List coloring was first studied by Vizing and by Erdos, Rubin, and Taylor. We investigate this problem considering several conditions under the list of available colors for each vertex: when the values are contiguous and the lower and upper bounds are known; when the lists have the same size or not; etc. For such restricted list coloring problems, we check the behavior of the choosability property for some classes of graphs. A graph is k-choosable (or k-list-colorable) if it has a proper list coloring no matter how one assigns a list of k colors to each vertex. The choosability (or list chromatic number) $\chi(G)$ of a graph G is the least number k such G is k-choosable. The choice number of G is equal to k if G is k-choosable but not $(k-1)$-choosable. In this work, implicit enumeration algorithms and heuristics for determining the choice number k of a graph, considering such restricted list coloring problems are also discussed.

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Acyclic edge coloring of the complete bipartite graph $K_{2p,2p}$

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Abstract

An *acyclic edge coloring* of a graph is a proper coloring of its edges in which the subgraph induced by any two colors has no cycles. The *acyclic chromatic index* of a graph G is the smallest integer k such that there is an acyclic edge coloring of G using k colors; it is denoted by $a'(G)$. It has been conjectured that $a'(G) \leq \Delta + 2$, for any G. Let $K_{n,n}$ denote the complete bipartite graph with independent sets of size n. The only values of n for which it is known that $a'(K_{n,n}) \leq n + 2$, are when $n \in \{p, p^2, 2p - 1\}$, for a prime p. In this work we improve on the best known generic upper bound $a'(K_{n,n}) \leq 5n$, when $n = 2p$ and p is prime. We prove that $a'(K_{2p,2p}) \leq 2p + 4$, for each p prime. The construction used to color the graph is based on a one-factorization of $K_{2p,2p}$ with p^2 pairs of perfect matchings inducing a Hamiltonian cycle. This latter construction is extended to each $K_{n,n}$, with n even, showing that $pf(n) = \frac{n^2}{4}$, where $pf(n)$ is the maximum over all one-factorizations $\mathcal{F}$ of $K_{n,n}$ of the number of pairs of perfect matchings in $\mathcal{F}$ inducing a Hamiltonian cycle in $K_{n,n}$.

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Forbidden subgraph characterization of star directed path graphs that are not rooted directed path graphs

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Gavril proved that a graph G is *chordal* if and only if there is a tree T, called *clique tree, or model*, whose vertices are the cliques of the graph and for every vertex x of G the cliques that containing x induce a subtree in the tree which we will denote by T_x. Natural subclass of chordal graphs are directed path graphs and rooted directed path graphs. A graph G is a *directed path graph* (respectively *rooted directed path graph*) if it there exists a model T that can be oriented (respectively oriented and rooted) and such that T_x is a oriented subpath of T for every $x \in V(G)$.

Panda presented the characterization of directed path graphs by forbidden induced subgraphs.

Characterizing rooted directed path graphs by forbidden induced subgraphs is an open problem. It is certainly too difficult a characterization of this class by forbidden induced subgraphs as there are too many (families of) graphs to exclude but in [1] was proposed a conjecture to characterize rooted directed path graph. In this original form this conjecture is not complete but in [3] was proved on directed path graphs with leafage four having two minimal separators which has multiplicity two.

A graphs is a *star directed path graph* if it is a directed path graph and has a directed path model that is a star. This class of graphs contains directed path graphs that are split and it has intersection with directed path graphs with leafage four having two minimal separators which has multiplicity two. In this work, we prove that minimal star directed path graphs that are not rooted path graphs has leafage four. Thus we build the family of forbidden for this class.

References

[1] K. Cameron, C. T. Hoáng, B. Lévêque, Asteroids in rooted and directed path graphs, *E*lectronic Notes in Discrete Mathematics **32** (2009), 67-74.

[2] K. Cameron, C. T. Hoáng, B. Lévêque, Characterizing directed path graphs by forbidden asteroids, *J*ournal of Graph Theory **68** (2011), 103–112.

[3] M. Gutierrez, S. B. Tondato, Special asteroidal quadruple on directed path graph non rooted path graph, *E*lectronic Notes in Discrete Mathematics **44** (2013), 47-52.

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On the class $[h, 2, 2]$ local

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An **(h,2,2)-representation** of a graph G is a pair $\langle \mathcal{P}, T \rangle$ where T is a tree with maximum degree h and $\mathcal{P}$ is a family $(P_v)_{v \in V(G)}$ of subpaths of T satisfying that two vertices v and v' of G are adjacent if and only if P_v and $P_{v'}$ have at least two vertices (one edge) in common. We let **[h,2,2]** denote the class of graphs which admit an $(h, 2, 2)$-representation. The well known class of EPT graphs (also called UE) is the union of the classes $[h, 2, 2]$ for $h \geq 2$. Determining the family of forbidden induced subgraphs for being EPT is an intricate open problem.

Our aim is to characterize by forbidden induced subgraphs the class $[h, 2, 2]$ for a fixed h. For this purpose, we search minimal structures in an EPT graph, that cannot be represented using a host tree with maximum degree h.

We define recursively the following family of graphs we have called *gates*:

(i) A chordless cycle C_n is a gate for every $n \geq 4$;

(ii) If G is a gate, C and C' are disjoint cliques (maximal complete subgraphs) of G, and $P : v_1,...,v_k$ is a chordless path disjoint from G with $k \geq 2$, then the union of G and P plus all edges between v_1 and the vertices of C, and all edges between v_k and the vertices of C' is a gate.

(iii) There are no more gates.

If the number of cliques of a gate G is h then we say that G is an h-gate.

In this work, we show that h-gates are EPT graphs that cannot be represented in a host tree with maximum degree less than h, this generalizes one of our previous results [1]. Even more, we conjecture that an EPT graph belongs to $[h, 2, 2]$ if and only if it has no induced subgraph isomorphic to a k-gate for $k > h$.

We also prove that the conjecture is true in a subclass of local-EPT graphs, this is the EPT graphs that admit a representation in which all paths of $\mathcal{P}$ share a vertex of T.

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[1] EPT graphs on bounded degree trees, Matemática Contemporánea. 42 (2014) 1-8.

On Restricted Multi-break Rearrangement and Sorting Separable Permutations

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A multi-break rearrangement represents most of genome rearrangements, as transpositions and reversals, a k-break cuts k adjacencies over a permutation, and forms k new adjacencies by joining the extremities according to an arbitrary matching [1]. Although the transposition and the reversal distances are both NP-hard problems [2, 3], the multi-break distance between two permutations is computed in polynomial time [1]. In this work we focus on the restricted multi-break rearrangement *rmb*, a restricted k break $rmb(a, b; c_1 \leftrightarrow d_1; \ldots; c_k \leftrightarrow d_k)$ inverts the block from position a to b, maintaining the order from position c_1 to d_1, ..., and from position c_k to d_k.

A cograph is a P_4-free graph, subclass of the permutation graph. The permutations that characterize the cographs are the separable permutations, exactly the permutations which do not contain neither $[2\,4\,1\,3]$ nor $[3\,1\,4\,2]$ as a pattern, i.e. do not contain a subsequence of four elements whose relative order matches one of the two permutations above [4]. We study the sorting by restricted multi-break problem where we give a lower bound on the *rmb* distance, and we give the exact distance for the separable permutations, considering the corresponding cotrees.

References

[1] Alekseyev, M. A., and Pevzner, P. A., Multi-break rearrangements and chromosomal evolution. Theor. Comput. Sci., 395(2), 193–202, (2008).

[2] Bulteau, L., Fertin, G., and Rusu, I., Sorting by transpositions is difficult, SIAM J. Discrete Math., 26(3), 1148–1180, (2012).

[3] Caprara, A., Sorting permutations by reversals and Eulerian cycle decompositions. SIAM J. Discrete Math., 12(1), 91–110, (1999).

[4] Stankova, Z., E., Forbidden subsequences, Discrete Math., 132, 291–316, (1994).

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On the b-continuity of graphs

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A b-coloring of a graph is a (proper) coloring of its vertices such that for each color class there is at least one vertex that is adjacent to all possible colors. A k-b-coloring is a b-coloring on k colors.

One peculiar characteristic of b-colorings is that for some graphs there is an integer k such that G has both a $k-1$ and a $(k+1)$-b-coloring but does not have any k-b-colorings. Otherwise, G is called a b-continuous graph.

In this talk I will survey results concerning b-continuity of graphs belonging to special graph classes.

Cliques, Coloring and Satisfiability: from structure to algorithms

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Cliques, coloring and satisfiability are three central problems of theoretical computer science each of which is generally $\mathcal{NP}$-hard. On the other hand, each of them may become tractable when restricted to instances of particular structure. In this talk we analyze how the structure of the input can affect the computational complexity of these problems. We also discuss some algorithmic tools to solve the problems with a focus given to transformations of graphs and of CNF formulas.

Periphery and convexity of a graph*,†

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Let $G = (V(G), E(G))$ be a finite, simple and connected graph. Given a set $S \subseteq V(G)$, we say that S is *geodetic* if the set of vertices lying on shortest paths between any pair of vertices of S is equal to $V(G)$. We say that S is *monophonic* if the set of vertices lying on an induced path between any pair of vertices of S is equal to $V(G)$. The *eccentricity* of a vertex v is the number of edges in the greatest shortest path between v and any vertex w of G. The *diameter* $diam(G)$ of a graph G is the maximum eccentricity of the vertices in $V(G)$. The *periphery* $Per(G)$ of G is the set formed by vertices v such that the eccentricity of v is equal to the diameter of G. The *contour* of G is the set of vertices v such that no neighbor of v has an eccentricity greater than v.

The problem of determining whether the contour of a graph class is geodetic was proposed in 2005 by Cáceres et al.. After this, many papers were published about this subject. In some of them the authors investigated the problem for specific classes of graphs. In 2013 Artigas et al. established a condition for the contour of a graph G to be geodetic in terms of the diameter of G. In this work we extend these results for the periphery of a graph. We also consider the problem of deciding whether $Per(G)$ is a monophonic set. We remark that the contour of a graph is always monophonic.

We show that if $diam(G) \leq 2$ then $Per(G)$ is geodetic, and $Per(G)$ is not necessarily geodetic if $diam(G) > 2$. Particularly, we characterize the graphs G with $diam(G) = 3$ such that $Per(G)$ is not geodetic. These results lead us to solve the problem for classes of graphs like cographs, chordal, split and threshold graphs. We show that the same conditions do not generalize for graphs with diameter equal to 4. We prove that if G is a power of a path, then $Per(G)$ is a geodetic set, and show a infinite family of unit interval graphs for which the periphery is not geodetic. In addition, we describe graphs, for which the contour is geodetic and the periphery is not a geodetic set. Finally, we consider the problem of determining whether $Per(G)$ is a monophonic set.

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On the l-neighborhood convexity

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Let G be a finite, simple, and undirected graph and let S be a set of vertices of G. We say that a set $S \subseteq V(G)$ is a convex set if every vertex in $V(G) \setminus S$ has less than l neighbors in S. The *convex hull* $H_G(S)$ of S is the smallest convex set containing S. A *hull set* of G is a set of vertices whose convex hull equals the whole vertex set of G, and the minimum cardinality of a hull set of G is the *hull number* $h(G)$ of G. Finally, the *Carathéodory number* of G is the smallest integer c such that for every set S and every vertex u in $H_G(S)$, there is a set $F \subseteq S$ with $|F| \leq c$ and $u \in H_G(F)$.

In [1], it has been determined the Carathéodory number of trees and in [2] it has been determined the hull number of the cographs, both, in the 2-neighborhood convexity.

In this work, we study the hull number and the Carathéodory number for the l-neighborhood convexity of graphs considering $l > 2$. We determine the hull number for cographs in the 3-neighborhood convexity and determine the Carathéodory number of trees for l-neighborhood convexity, where $l > 2$.

References

[1] R. M. Barbosa, E. M. M. Coelho, M. C. Dourado, D. Rautenbach and J. L. Szwarcfiter, *On the Carathéodory number for the convexity of paths of order three*, SIAM Journal on Discrete Mathematics, v. 26, p. 929-939, 2012.

[2] C. C. Centeno, M. C. Dourado, J. L. Szwarcfiter, *On the Convexity of Paths of Length Two in Undirected Graphs*, Eletronic Notes in Discrete Mathematics, 32, p. 11-18, 2009.

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A TIGHT UPPER BOUND FOR THE HELLY NUMBER OF THE GEODETIC CONVEXITY ON BIPARTITE GRAPHS

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In this work we consider finite, simple and undirected graphs. Consider a graph G. In the geodetic convexity on G, a set of vertices $S \subseteq V(G)$ is *(geodesically) convex* if the vertices of any shortest path joining the vertices of S is contained in S. The *(geodetic) convex hull of* S, $H(S)$, is the smallest convex set containing S. In the monophonic convexity these concepts are defined similarly by simply replacing "shortest paths" by "induced paths" in the definition of convex sets.

The *core* of a family of sets is the total intersection of the sets of this family. A family of sets $\mathcal{F}$ is k*-intersecting* if the core of every k sets of $\mathcal{F}$ has non-empty core. A family of sets $\mathcal{F}$ is k*-Helly* if every k-intersecting subfamily of $\mathcal{F}$ has non-empty core. The *Helly number* of $\mathcal{F}$ is the smallest number h such that $\mathcal{F}$ is h-Helly. The Helly number of the convexity $\mathcal{C}$ on a graph G is the Helly number of the family defined by the convex sets of the convexity $\mathcal{C}$ on G.

A survey on computational aspects of the Helly number can be found in [1].

In [2] it was shown that the Helly number of the monophonic convexity on any graph G is equal to the size of the maximum clique of G.

In this work we present an upper bound for the Helly number of the geodetic convexity on bipartite graphs and construct an infinity family of graphs $\mathcal{G} = \{G_1, \ldots, G_k, \ldots\}$, where $G_i \in \mathcal{G}$ is an example of a bipartite graph with Helly number i in the geodetic convexity reaching the bound, therefore showing that the bound is tight.

References

[1] M.C. Dourado and F. Protti and J.L. Szwarcfiter, Computational aspects of the Helly property: Graphs and hypergraphs, *Electron. J. Combin.* Dynamic survey number 17, 2009.

[2] R.E. Jamison and R.J. Nowakowski, A Helly theorem for convexity in graphs, *Discrete Mathematics*, 35–39, 1984.

[3] I.M. Pelayo, *Geodesic Convexity in Graphs*, Springer New York, 2013.

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On the diameter of the Cayley Graph $H_{l,p}$

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In this work, our interests are in the design and analysis of static networks. Static networks can be modeled using tools from Graph Theory. The graph is the interconnection network, the processors are the vertices and the communication links between processors are the edges connecting the vertices. There are several parameters of interest to specify a network: low degree, low diameter, and the distribution of the node disjoint paths between a pair of vertices in the graph. The degree relates to the port capacity of the processors and hence to the hardware cost of the network. The maximum communication delay between a pair of processors in a network is measured by the diameter of the graph. Thus, the diameter is a measure of the running cost.

The definition of Cayley graphs was introduced to explain the concept of abstract groups which are described by a generating set. The Cayley graphs are regular, may have logarithmic diameter, are maximally fault tolerant and have a rich variety of algebraic properties. One such algebraic property is that Cayley graphs are vertex transitive, i.e., the graph looks the same when viewed from any vertex. One important consequence of the vertex transitivity is that a guest structure embedded in one region of the host network can be readily translated to another region without affecting the quality of the original embedding.

The family $H_{l,p}$ has been defined in the context of edge partitions, subsequently shown to be composed by Hamiltonian Cayley graphs, and after we showed an algorithm to calculate the diameter of graph $H_{l,p}$ of time $O(l)$. The established properties support the graph $H_{l,p}$ to be good schemes of interconnection networks. We consider families of Cayley graph $H_{l,p}$. The p^{l-1} vertices of the graph $H_{l,p}$ are the l-tuples with values between 0 and $p-1$, such that the sum of the l values is congruent to 0 $mod\ p$, and there is an edge between two vertices having two corresponding pairs of entries whose values differ by one unit. Our goal is to find the diameter of Cayley graph $H_{l,p}$ with complexity $O(log(l+p))$. In this work, we present new results on the diameter $D = \frac{p \cdot l}{4}$ of the Cayley graph $H_{l,p}$ when l and p are even.

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Grafos de Permutação Redutíveis Canônicos: caracterização, reconhecimento e aplicação a marcas d'água digitais*

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Um grafo de fluxo redutível $G = (V, E, s)$ é um grafo direcionado com uma fonte $s \in V(G)$, tal que, para cada ciclo C de G, todo caminho direcionado de s a C chega a C pelo mesmo vértice de C. Diversas pesquisas na área de proteção de software desenvolvidas recentemente estão relacionadas a uma subclasse dos grafos de fluxo redutíveis, chamada de *grafos de permutação redutíveis* [1, 2]. Tais grafos possuem, entre outras características, caminho hamiltoniano único.

Neste trabalho, apresentamos uma caracterização de uma subclasse dos grafos de permutação redutíveis, chamada *grafos de permutação redutíveis canônicos*. Como consequência desta caracterização, que é baseada em propriedades estruturais, obtivemos um algoritmo linear de reconhecimento. Grafos de permutação redutíveis canônicos podem ser utilizados para codificar marcas d'água digitais, e correspondem de fato aos grafos gerados pelo algoritmo de codificação de marcas d'água apresentado em [2]. Além da caracterização e do reconhecimento de tais grafos, apresentamos um algoritmo polinomial que recupera, sempre que possível, um grafo da classe com um número constante de arestas removidas, e também um algoritmo linear para restaurar grafos de permutação redutíveis canônicos com até duas arestas removidas — o que provamos ser sempre possível.

References

[1] Christian Collberg and Stephen Kobourov and Edward Carter and Clark Thomborson, Graph-Based Approaches to Software Watermarking *29th Workshop on Graph-Theoretic Concepts in Computer Science, WG'03.* LNCS **2880** (2003), 156–167.

[2] Maria Chroni and Stavros D. Nikolopoulos, An Efficient Graph Codec System for Software Watermarking *36th IEEE Conference on Computers, Software and Applications (COMPSAC'12).* IEEE Proceedings (2012), 595–600.

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Sistemas modulares de dígitos verificadores ótimos

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Neste trabalho discutimos vários sistemas de dígitos verificadores utilizados no Brasil, muitos deles semelhantes a esquemas usados mundialmente [3, 4], e fazemos uma análise da sua capacidade de detectar os diversos tipos de erros que são comuns na entrada de dados em sistemas computacionais. A análise nos mostra que os esquemas escolhidos constituem decisões subotimizadas e quase nunca obtêm a melhor taxa de detecção de erros.

Os sistemas de dígitos verificadores são baseados em três teorias da álgebra [1, 2]: a-ritmética modular, teoria de grupos e quasigrupos. Focamos o estudo nos sistemas baseados em aritmética modular, para os quais apresentamos várias melhorias que podem ser introduzidas. Desenvolvemos um novo esquema ótimo baseado em aritmética modular base 10 com três permutações que utiliza um dígito verificador para identificadores numéricos de tamanho maior do que sete. Para identificadores de tamanho até sete, descrevemos o esquema Verhoeff, já antigo, mas pouquíssimo utilizado e que também é uma alternativa de melhoria.

Desenvolvemos ainda, esquemas ótimos para qualquer base modular prima que detectam 100% dos tipos de erros considerados. Estes podem ser utilizados para identificadores alfanuméricos ou identificadores numéricos utilizando dois dígitos verificadores. Fazemos uso ainda de elementos da estatística, no estudo das probabilidades de detecção de erros e de algoritmos, na obtenção de esquemas ótimos.

References

[1] H. M. Damm. Check digit systems over groups and anti-symmetric mappings. *Archiv Der Mathematik*, 75(6):413–421, 2000.

[2] J. A. Gallian. *Contemporary Abstract Algebra*, volume 70 ed. Cengage Learning, 2010.

[3] J. A. Gallian and S. Winters. Modular arithmetic in the marketplace. *The American Mathematical Monthly*, 95(6):548–551, 1988.

[4] J. Verhoeff. *122 f. Thesis, Mathematical Centre Tract 29.* The Mathematical Centre, 1969.

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The Biclique Graph of some classes of graphs II

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A biclique of a graph is a vertex set that induces a maximal complete bipartite subgraph. The biclique graph of a graph G, denoted by $K_B(G)$, is the intersection graph of the bicliques of G.

For a graph class $\mathcal{C}$, $K_B(\mathcal{C})$ denotes the family of biclique graphs of graphs in $\mathcal{C}$.

In this work we studied $K_B(\mathcal{C})$ for some graph classes: split graphs, threshold graphs, gem-free split graphs and bipartite permutation graphs. We seek for properties, characterizations, recognition algorithms and/or determine the computational complexity of the recognition problem. That is, given a graph G we ask if there exists a graph H in $\mathcal{C}$ such that $G = K_B(H)$.

When $\mathcal{C}$ is the class of split graphs we proved that every graph $G \in K_B(\mathcal{C})$ has diameter less or equal to 2; the connectivity (and edge-connectivity) of G is greater or equal to $2(n-2)$, where n is the number of vertices of the complete part of the split graph; and G is hamiltonian. Also, every graph H is an induced subgraph of some graph of $K_B(\mathcal{C})$, which implies that $K_B(\mathcal{C})$ has no forbidden induced subgraphs.

For the threshold graphs and gem-free split graphs (which are subclasses of split graphs), we found polynomial time algorithms for the recognition problem of the class $K_B(\mathcal{C})$.

At the moment we are working on the case where $\mathcal{C}$ is the class of bipartite permutation graphs. We proved that $K_B(\mathcal{C}) \subset$ interval graphs and every graph in $K_B(\mathcal{C})$ is hamiltonian. We also found some forbidden subgraphs and partial characterization.

On the other hand, we studied the inverse problem, $K_B^{-1}(\mathcal{C})$: given a graph G we want to know if $K_B(G)$ belongs to $\mathcal{C}$.

We proved that, for every $\mathcal{C}$ that contains the complete graphs and is C_4-free, the problem of determining if $K_B(G)$ is in $\mathcal{C}$ is in co-NP-complete.

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Maximum Clique via MaxSat and Back Again

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Among the exact algorithms proposed for the Maximum Clique Problem in the literature, branch and bound schemes stand out as the best performing from an experimental point of view. Moreover, authors seem to agree in that the best choice for the bounding function in such algorithms is the use of (an upper bound on) the chromatic number of the graph. Indeed, it is remarkable that the main recent advances on the subject essentially dwell on the same algorithm, varying only in the proposed way to color the graph.

Such is the focus also in [1], where it is proposed the idea of refining the bound given by coloring via a rather unusual reduction to the Maximum Satisfiability Problem (MaxSat), which is the problem of maximizing the number of satisfied clauses in a given boolean formula. The authors report the experimental performance of their implementation and, based on them, conclude that the approach *"is a very promising research direction"*.

Schematically their idea may be described as follows. Given a graph G and a coloring of G, an instance of MaxSat is computed. This instance is then given as input to a certain heuristic algorithm for MaxSat. If this heuristic is able to output an upper bound on the number of satisfiable clauses of the MaxSat instance, this bound is then translated into an upper bound for the size of the maximum clique in G, and this bound is lower than the number of colors in the original coloring.

While the reduction proposed in [1] is not particularly complicated or unnatural, the fact that the algorithm is described in terms of propositional calculus obscures its graph theoretic meaning. We show that, although the main novelty presented in [1] is the use of "MaxSat technology", their idea can be expressed in pure graph theoretical terms, and that such description has several advantages. It leads to an algorithm which is shorter and simpler to describe and resorts only to usual graph theoretic concepts. Moreover, the resulting algorithm is a natural one in the sense that no artificial constructions or formulations are needed when the heuristic for MaxSat is interpreted back in graph theoretic terms. As such, the idea can be used as a starting point for further refinements and, last but not least, the simplification of the algorithm simplifies its analysis.

References

[1] Chu-Min Li and Zhe Quan. An efficient branch-and-bound algorithm based on maxsat for the maximum clique problem. In *Twenty-Fourth AAAI Conference on Artificial Intelligence*, 2010.

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Clique and neighborhood characterizations of strongly chordal graphs

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Maximal cliques and closed neighborhoods sometime play almost interchangeable roles in graph theory. For instance, interchanging them makes two existing characterizations of chordal graphs into two new characterizations. More intriguingly, these characterizations of chordal graphs can be naturally strengthened to new characterizations of strongly chordal graphs.

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AVD-TOTAL-COLOURING OF SOME FAMILIES OF SNARKS*

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Let $G := (V(G), E(G))$ be a simple graph. A *k-total-colouring* ϕ of G is a mapping $\phi\colon V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ such that any pair of adjacent vertices or adjacent edges receive distinct colours, and any pair of incident elements receive distinct colours. Let $C(u) = \{\phi(u)\} \cup \{\phi(uv)\colon uv \in E(G)\}$ be the set of colours that *occur* in a vertex $u \in V(G)$. If $C(u) \neq C(v)$ for any pair of adjacent vertices $u, v \in V(G)$, then ϕ is an *adjacent-vertex-distinguishing-total-colouring (AVD-total-colouring)* of G. The *adjacent-vertex-distinguishing-total-chromatic number (AVD-total-chromatic number)* of G, denoted $\chi''_a(G)$, is the smallest number of colours for which G admits an AVD-total-colouring.

In 2005, Zhang et al. [3] introduced AVD-total-colourings and conjectured that any simple graph G admits an AVD-total-colouring with at most $\Delta(G) + 3$ colours. Although this conjecture remains open for arbitrary graphs, it has been proven for some families of graphs, such as simple graphs with maximum degree three [1, 2]. Later, J. Hulgan [2] conjectured that any simple graph G with $\Delta(G) = 3$ admits an AVD-total-colouring with at most five colours.

An important class of 3-regular graphs is the class of snarks. *Snarks* are 3-regular graphs with chromatic index equal to four, and that do not have a cut edge. These graphs had their origin in the search for counterexamples to the Four Colour Theorem. In this work, we prove that the following families of snarks admit an AVD-total-colouring with exactly five colours: flower-snarks, Goldberg snarks, Blanuša generalized snarks and a family of Loupekine snarks. These results reinforce Hulgan's conjecture.

References

[1] X. Chen. On the adjacent vertex distinguishing total coloring numbers of graphs with $\Delta = 3$. *Discrete Mathematics*, 308:4003–4007, 2008.

[2] J. Hulgan. *Graph coloring with constraints.* PhD thesis, Mathematical Sciences, University of Memphis, May 2010.

[3] Z. Zhang, X. Chen, J. Li, B. Yao, X. Lu and J. Wang On adjacent-vertex-distinguishing total colouring of graphs. *Science in China, Series A: Mathematics*, 48(3):289–299, 2005.

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The Total Coloring of the 3rd and 4th Powers of Cycles

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A *total coloring* of a graph G is an assignment of colors to the edges and vertices of G, so that adjacent or incident elements have different colors. The *total chromatic number* of a graph G, denoted $\chi''(G)$, is the least number of colors for which G has a total coloring. It is easy to see that $\chi''(G) \geq \Delta(G)+1$ for every simple graph G with maximum degree $\Delta(G)$. To decide whether $\chi''(G)$ equals $\Delta(G)+1$ is NP-complete. Even when the problem is restricted to determining the total chromatic number of graphs in some particular class, as bipartite k-regular graphs for each fixed $k \geq 3$, it is NP-hard. The famous Total Coloring Conjecture (TCC) states that a graph G with maximum degree $\Delta(G)$ has $\chi''(G) \leq \Delta(G)+2$. When a simple graph G has $\chi''(G) = \Delta(G)+1$, it is called *type 1* and when $\chi''(G) = \Delta(G)+2$, it is called *type 2*.

The *k-th power of* G, denoted by G^k, is the graph with the same vertex set as G and where two vertex are adjacent if, and only if, their distance in G is at most k. The k-th power of a cycle with n vertices is denoted by C_n^k. When k equals 1, C_n^1 is isomorphic to C_n and it is known that C_n is type 1 if $n \equiv 0 \pmod 3$, and type 2 otherwise. If C_n^k has $k \geq \lfloor \frac{n}{2} \rfloor$, then C_n^k is isomorphic to the complete graph K_n, which is type 1 when n is odd, and type 2 when n is even. Campos and de Mello showed that $\chi''(C_n^k) \leq \Delta(C_n^k)+2$, satisfying the TCC. They conjectured that the C_n^k, $2 \leq k < \lfloor \frac{n}{2} \rfloor$, is type 2 when $k > \frac{n}{3} - 1$ and n is odd; and type 1 otherwise. They also proved that this conjecture is true for C_n^2.

In this work, we show that the Conjecture of Campos and de Mello is true for C_n^3 and C_n^4. For each $0 \leq r \leq 2k$ and each $k \in \{3,4\}$, we present a total coloring for the minimum type 1 power of cycle C_n^k with $r \equiv n \pmod{2k+1}$. We show how to use this coloring to polinomially construct an optimal total coloring for any other power of cycle with the same k and with n' vertices, when $n' > n$ and $n' \equiv r \pmod{2k+1}$. In order to achive this goal, we use special total colorings for powers of paths P_a^k with $a \geq 2k+1$. Considering the general case, if a total coloring with $2k+1$ colors is given to a C_n^k, and a special total coloring is given to the power of path P_{2k+1}^k, then the same technique can be applied to obtain a total coloring for any other power of cycle $C_{n'}^k$, with $n' > n$ and $n' \equiv n \pmod{2k+1}$.

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Complexity of the oriented coloring in planar, cubic oriented graphs

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It has been a challenging problem to determine the smallest graph class where a problem is proved to be hard. In the literature, this has been pointed out to be very important in order to establish the real nature of a combinatorial problem.

An *oriented k-coloring* of an oriented graph $\vec{G} = (V, \vec{E})$ is a partition of V into k subsets such that there are no two adjacent vertices belonging to the same subset, and all the arcs between a pair of subsets have the same orientation. The decision problem k-ORIENTED CHROMATIC NUMBER ($\textsc{ocn}_k$) consists of an oriented graph $\vec{G}$ and an integer $k > 0$, plus the question if there exists an oriented k-coloring of $\vec{G}$. By its strong appeal, many papers have presented NP-completeness proofs for $\textsc{ocn}_k$. We noticed that it was not known the complexity status of $\textsc{ocn}_k$ when the input graph $\vec{G}$ satisfies that the underlying graph G is cubic.

In this work we prove using [1] that $\textsc{ocn}_4$ is NP-complete for an oriented graph $\vec{G}$ such that G is at same time planar and cubic.

Our result defines a P versus NP-complete dichotomy with respect to the degree of G: $\textsc{ocn}_k$ is polynomial if $\Delta < 3$ and NP-complete if G is cubic and $\Delta \geq 3$ [1], since it is known that $\textsc{ocn}_3$ is in P [2], and that $\textsc{ocn}_k$ is in P when the underlying graph has $\Delta \leq 2$.

References

[1] H. Coelho, L. Faria, S. Gravier and S. Klein, Oriented coloring in planar, bipartite, bounded degree 3 acyclic oriented graphs, *ENDM* **44** (2013) 195–200.

[2] W. F. Klostermeyer and G. MacGillivray, Homomorphisms and oriented colorings of equivalence classes of oriented graphs, *Disc. Math.* **274** (2004) 161-172.

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A characterization of PM-compact bipartite and near-bipartite graphs

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The perfect matching polytope of a graph G is the convex hull of the incidence vectors of all perfect matchings in G. In this talk, we characterize bipartite and near-bipartite graphs whose perfect matching polytopes have diameter 1 (it is a clique).

Total coloring of snarks is NP-complete

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A *k-total-coloring* of G is an assignment of k colors to the edges and vertices of G, so that adjacent or incident elements have different colors. The *total chromatic number* of G, denoted by $\chi''(G)$, is the least k for which G has a k-total-coloring. The well-known Total Coloring Conjecture states that $\Delta + 1 \leq \chi'' \leq \Delta + 2$ and has been proved for cubic graphs [3]. Hence, the total chromatic number of a cubic graph is either 4, in which case the graph is called *Type 1*, or 5, in which case it is called *Type 2*. *Snarks* are bridgeless cubic graphs that do not allow a 3-edge-coloring, and their importance arises at least in part from the fact that several well-known conjectures would have snarks as minimal counterexamples. Motivated by the question proposed by Cavicchioli et al. [1] of finding, if one exists, the smallest Type 2 snark of girth at least 5, we investigate the total coloring of snarks.

It is shown in [4] that the problem of determining if a cubic bipartite graph has a 4-total-coloring is NP-complete. We prove that, similarly, the problem of determining if a snark is Type 1 is NP-complete. Our proof resembles the one in [4] but requires a slightly different construction. The proof is by reduction from the NP-complete problem of determining if a 4-regular graph has a 4-edge-coloring [2]. More specifically, given a 4-regular graph G, we show that it is possible to construct a snark G^P by replacing each vertex of G by a certain graph P which has special coloring properties. This construction can be done in polynomial time in the order of G, and we prove that G has a 4-edge-coloring if and only if G^P has a 4-total-coloring.

References

[1] A. Cavicchioli, T. E. Murgolo, B. Ruini, F. Spaggiari. Special classes of snarks. *Acta Appl. Math.*, **76**, (2003) pp. 57–88.

[2] D. Leven, Z. Galil. NP-completeness of finding the chromatic index of regular graphs. *J. Algorithms*, **4**, (1983) pp. 35–44.

[3] M. Rosenfeld. On the total coloring of certain graphs. *Israel J. Math.*, **9**, (1971) pp. 396–402.

[4] A. Sánchez-Arroyo. Determining the total colouring number is NP-hard. *Discrete Math.*, **78**, (1989) pp. 315–319.

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Complexity between Domination, Independent, Connected, and Paired Domination Numbers

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We consider finite, simple, and undirected graphs and use standard terminology and notation. Let D be a set of vertices of some graph G. The set D is a *dominating set of* G if every vertex of G that does not belong to D, has a neighbour in D. The set D is an *independent dominating set of* G if it is a dominating and an independent set of G. The set D is a *connected dominating set of* G if it is dominating and the graph $G[D]$ is connected. Finally, the set D is a *paired dominating set of* G if it is dominating and the graph $G[D]$ has a perfect matching. The *domination number* $\gamma(G)$, the *independent domination number* $\iota(G)$, the *connected domination number* $\gamma_c(G)$, and the *paired domination number* $\gamma_p(G)$ of G are the minimum cardinalities of a dominating, an independence dominating, a connected dominating and a paired dominating set of G, respectively. These definitions immediately imply

$$\gamma(G) \leq \iota(G) \tag{1}$$
$$\gamma(G) \leq \gamma_c(G) \tag{2}$$
$$\gamma(G) \leq \gamma_p(G) \tag{3}$$

for every graph G where the parameters are well defined.
Very little is known about extremal graphs, that is, the graphs that satisfy one of these inequalities with equality. So it is usual to work with a less complex class. In that sense, V.E. Zverovich, 1995, I.E. Zverovich, 2003, and J.D. Alvarado et al., 2014, studied classes of graphs defined by requiring equality in (1), (2), or (3), respectively, for all induced subgraphs (where the parameters are well defined). Their results are characterizations of these classes in terms of their minimal forbidden induced subgraphs.
In this work, we prove the following hardness results, which suggest that the extremal graphs for some of the above inequalities do most likely not have a simple description.

Theorem 1 *It is NP-hard to decide, for a given graph G, whether $\gamma(G) = \iota(G)$.*

Theorem 2 *It is NP-hard to decide, for a given graph G, whether $\gamma(G) = \gamma_c(G)$.*

Theorem 3 *It is NP-hard to decide, for a given graph G, whether $\gamma(G) = \gamma_p(G)$.*

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Aliança Global Ofensiva em Alguns Produtos Lexicográficos em Grafos

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Uma *aliança ofensiva* em um grafo G é um conjunto A de vértices com a propriedade que todos os vértices não pertencentes a A tem um ou mais vizinhos em A com relação aos que estão fora de A. Este e outros conceitos sobre alianças foram apresentados inicialmente por Hedetniemi, Hedetniemi e Kristiansen em 2004, [1]. A complexidade computacional e aplicações para a defesa nacional, redes de computadores, distribuição computacional e redes sociais são exemplos que motivam os estudos sobre alianças em grafos. Uma *aliança ofensiva* A, é *global* se for também um conjunto dominante de G, ou seja, se todos os vértices de G e não pertecentes a A tem pelo menos um vizinho em A. O *número da aliança ofensiva global* de G, $\gamma_o(G)$, é a cardinalidade mínima de uma aliança ofensiva global de G. Denotamos o produto lexicográfico de dois grafos G e H por $G \circ H$. Yero e Rodriguez-Velázquez, em [2] apresentaram alguns resultados para aliança ofensiva global de algumas famílias de produto Cartesiano de grafos. Neste trabalho nós estabelecemos alguns limites inferiores para produto lexicográfico de caminhos, ciclos e grafos estrela, em particular encontramos os valores para $\gamma_o(P_n \circ C_m)$, $\gamma_o(C_n \circ C_m)$, $\gamma_o(P_n \circ S_m)$ e $\gamma_o(C_n \circ S_m)$, onde $n, m \geq 3$.

Referências

[1] S.M. Hedetniemi, S.T. Hedetniemi and P. Kristiansen, *Alliances in graphs*, J. Combin. Math. Combin. Comput.48 (2004), 157-177.

[2] I. G. Yero and J. A. Rodriguez-Velázquez, *Computing global offensive alliances in Cartesian product graghs*, Discrete Applied Mathematics.161(2013), 284-293.

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The Burning of the Snarks

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The Firefighter game was introduced by Hartnell at the 25th Manitoba Conference on Combinatorial and Computing in Winnipeg (1995). It is a model containment of the spreading of an undesired property within a network, like an infecting disease. Let (G, r) be a pair, where G is simple, undirected, and connected rooted graph with root $r \in V(G)$. The game proceeds in rounds, and at round 0, a fire breaks out at vertex r. In subsequence rounds, the firefighter *defends* at most one vertex, which is not burned and not defended in previous rounds; and so, the fire spreads on all vertices of G that are neither burned nor defended, and have a burned neighbour. Once burned or defended, a vertex remains so for the rest of the game. The process ends when the fire can no further spread. The objective is to choose a sequence of vertices (*strategy*) for the firefighter to protect, so as to save the maximum number of vertices in the graph.

FIREFIGHTER PROBLEM:

Instance: A rooted graph (G, r) and an integer $k \geq 1$.

Question: If the fire breaks out at r, is there a strategy under which at most k vertices burn?

Finbow et al. (2007) showed that the FIREFIGHTER PROBLEM is NP-complete for trees of maximum degree three, and presented a tractable case on a graph of maximum degree three when the fire breaks out at vertex of degree two. This implies that the FIREFIGHTER PROBLEM is NP-complete for any graph of maximum degree three such that the fire breaks out in a vertex of degree three. Hence, a natural question arises: what is the structure of graphs with degree at most three such that the FIREFIGHTER PROBLEM has polynomial complexity?

We studied the FIREFIGHTER PROBLEM on snarks. The definition of *snarks* was motivated by the search of counter-examples to the graph four-color conjecture. The importance of these graphs remains so far from the fact that several relevant conjectures stated in the past would have snarks as minimal counter-examples. The *surviving rate* $\rho(G)$ of a graph G with order n is defined to be the average proportion of vertices that can be saved when a fire randomly breaks out at a vertex of G. In this work,we show a lower bound for the surviving rate for Flower, Goldberg, and Blanusa snarks.

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Timber Game with Caterpillars*

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Timber is a two player game introduced by Nowakowski et al. in 2013. The game timber is played on a directed graph $D = (V, \vec{E})$, with a domino on each edge. The player chooses a domino on some edge, say $\vec{xy}$ and topples it in the direction of y. (This is the only time that the direction of the edge is important). This domino topples the dominoes on the edges incident to y, independent of whether the edge is directed into or away from y, and the process of toppling the dominoes continues until no more dominoes topple. The toppled dominoes and the corresponding edges are removed from the digraph.

Let $G = (V, E)$ be a graph. A *configuration* $D = (V, \vec{E})$ is an orientation of G. The orientation of the arc represents the available movement of the domino piece. When we play with piece $\vec{xy}$, each adjacent path that has y as one of its extreme vertices will be excluded from the digraph, regardless of the orientation of its edges. All other vertices remain in the digraph.

A *P-position* is a configuration D in which the second player wins, independent of what the first player plays. The number of P-positions of a path is known. Our goal is to contribute to the open problem of determining the number of P-positions of a tree by studying the case of a caterpillar. A *caterpillar* $cat(k_1, k_2, \ldots, k_s)$ is a tree which is obtained from a central path $v_1, v_2, v_3, ..., v_s$ (called *spine*) by joining v_i to k_i new leaf vertices, $i = 1, \ldots, s$. Thus, the number of vertices is $n = s+k_1+k_2+\ldots+k_s$. Using this definition, a *caterpillar 1* is a $cat(1, \ldots, 1)$, i.e., $k_i = 1$, for all $i = 1, \ldots, s$; and a *double broom* is a $cat(k_1, 0, \ldots, 0, k_s)$, i.e., $k_i = 0$ for $i = 2, \ldots, s-1$. The *outdegree* of a vertex v is the number of arcs with v as their initial vertex. In this work, we show structural properties to determine whether a configuration D associated to a caterpillar is a P-position. More specifically, we show:

- Every digraph associated to a caterpillar that has a leaf with outdegree 1 and every digraph associated to a *caterpillar 1* have no associated P-position.
- The number of P-positions of $cat(k_1, k_2, \ldots, k_s)$ such that each k_i is even, $i = 1, \ldots, s$, is equal to the number of P-positions of a path with s vertices.
- Let $cat(k_1, 0, \ldots, 0, k_s)$ be a double broom. If k_1 and k_s are even and odd, respectively, then the number of P-positions is equal to the number of P-positions of a path with $s + 1$ vertices; and if k_1 and k_s are both odd, then the number of P-positions is equal to the number of P-positions of a path with $s + 2$ vertices.

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O PROBLEMA DA PARTIÇÃO EM CLIQUES DOMINANTES*

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Dado um grafo $G = (V(G), E(G))$, um conjunto $S \subseteq V(G)$ é denominado um *conjunto dominante* de G, se para todo vértice $v \in V(G)$, ou v é um elemento de S, ou v é adjacente a um elemento de S. O *número de dominação* de G, $\gamma(G)$, é a cardinalidade de um menor conjunto dominante de G. O *problema do conjunto dominante mínimo* consiste em determinar $\gamma(G)$ para um grafo G arbitrário e foi demonstrado ser NP-difícil em 1979, por M. Garey e D. Johnson. Este é um problema de grande importância teórica. Muitas aplicações podem ser modeladas como problemas de conjuntos dominantes e algumas delas levaram à definição de variantes do problema original. Uma destas variantes, consiste em adicionar a restrição de que o conjunto dominante seja uma clique, definindo desta forma uma *clique dominante.*

Em 1977, E. J. Cockayne e S. T. Hedetniemi[1] introduziram uma nova variante do problema original que tem atraído a atenção de vários pesquisadores. Uma *partição em conjuntos dominantes* de um grafo G é uma partição de $V(G)$ tal que cada uma de suas partes seja um conjunto dominante de G. O *problema da partição em conjuntos dominantes* consiste em determinar a cardinalidade máxima de uma tal partição. Uma extensão natural deste problema consiste em considerar partições em conjuntos dominantes de G com restrições adicionais. Em particular, o *problema da partição em cliques dominantes* consiste em determinar, caso exista, uma partição em conjuntos dominantes de G, de cardinalidade máxima, tal que cada uma de suas partes seja também uma clique. O *número clique dominativo* é definido como $d_{cl}(G) := \max\{|\mathscr{P}| : \mathscr{P}$ é partição em cliques dominantes de $G\}$.

Este trabalho aborda o problema da partição em cliques dominantes para algumas classes de grafos. Em particular, foram caracterizados os grafos bipartidos e as potências de ciclos que possuem partição em cliques dominantes, determinando os seus números clique dominativos, quando existem. Ainda neste trabalho, foram consideradas as operações de produto cartesiano e produto direto de grafos. Para o primeiro produto, foi demonstrado que apenas os grafos G obtidos pelo produto cartesiano de dois grafos completos K_p e K_q possuem partição em cliques dominantes e que $d_{cl}(G) = \max\{p, q\}$. Já para o segundo, foi demonstrado que o número clique dominativo do produto direto de dois grafos completos K_p e K_q é $\lfloor \frac{pq}{3} \rfloor$, $p, q \geq 3$.

Referências

[1] E. J. Cockayne and S. T. Hedetniemi, *Towards a theory of domination in graphs*, Networks, Vol 7, 247-261, 1977.

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Sobre posets representables mediante contención de caminos en un árbol

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Un *conjunto parcialmente ordenado o poset* $\mathbf{P}$ es un par (X, P) donde X es un conjunto no vacío y P es una relación binaria reflexiva, antisimétrica y transitiva definida en X; se escribe $x \leq y$ cuando $(x, y) \in P$. Si $x \in X$, el *conjunto descendente de* x es el conjunto $D(x) = \{y \in X : y < x \text{ en } P\}$.

Dado un poset $\mathbf{P} = (X, P)$ se dice que una familia de conjuntos $\mathcal{F} = (F_i)_{i \in I}$ es un *modelo por contención* de $\mathbf{P}$ si existe una función $f : I \to X$ biyectiva tal que para todo i y j en I se verifica que $F_i \subseteq F_j \leftrightarrow f(i) \leq f(j)$. Si los elementos de la familia $\mathcal{F}$ son intervalos de la recta real decimos que $\mathbf{P}$ es un poset CI, o bien que $\mathbf{P}$ admite un modelo por contención de intervalos [1]. Una generalización de los posets CI son los posets CPT definidos como aquellos que admiten un modelo por contención de caminos en un árbol [2]. Dado un poset $\mathbf{P} = (X, P)$ se llama *grafo de comparabilidad* de $\mathbf{P}$ al grafo simple $\mathbf{G_P} = (X, E)$ siendo $E = \{\{x, y\} : x < y\}$.

En este trabajo definimos a los *posets 2-tree* como aquellos posets cuyos grafos de comparabilidad son *grafos 2-tree* [3], damos una caracterización recursiva para esta clase de posets y demostramos que un poset *2-tree* $\mathbf{P}$ es CPT si y solo si para cada maximal m de $\mathbf{P}$ el subposet inducido en $\mathbf{P}$ por $D(m)$ es un poset CI.

References

[1] Dushnik, Ben and Miller, E.W.; *Partially Ordered Sets*, American Journal of Mathematics, vol.LXIII (1941).

[2] Golumbic, Martin Charles and Scheinerman Edward R.,*Containment Graphs, Posets, and Related Classes of Graphs*, Combinatorial Mathematics: Proceedings of the Third International Conference, Volume 555, 192–204 (1989).

[3] A. Brandstädt, V. B. Le, J. P. Spinrad, *Graph Classes: A Survey*, SIAM Monographs on Discrete Mathematics and Applications (1999).

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Kneser Graphs are Close to Being Hamiltonian*

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The vertices of the *Kneser graph* $K(n,k)$ are the k-subsets of $\{1,2,\ldots,n\}$ and two vertices are adjacent if the corresponding k-subsets are disjoint. For $n = 2k+1$, the Kneser graph $K(2k+1,k)$ is called the *odd graph* and it is denoted by O_k. The bipartite double graph of the Kneser graph $K(n,k)$ is known as the *bipartite Kneser graph* $B(n,k)$, whose vertices are the k-subsets, and $(n-k)$-subsets of $\{1,2,\ldots,n\}$ and the edges represent the inclusion between two such subsets. The graphs $K(n,k)$ and $B(n,k)$ are vertex-transitive and, therefore, they can provide a counterexample or more evidence to a long-standing conjecture due to Lovász which claims that every connected undirected vertex-transitive graph has a hamiltonian path.

It is well-known that the decision problem related to the hamiltonian cycle problem is NP-Complete. Thus, one recent trend is the search for related structures. In this aspect, having a hamiltonian prism in a graph is an interesting relaxation of being hamiltonian making such a graph "close" to being hamiltonian [3]. The *prism over a graph* G is the Cartesian product $G \square K_2$ of G with the complete graph on two vertices. Previously, the prism over $B(2k+1,k)$ was proved to be hamiltonian [2]. Later, the counterpart of this result was proved for O_k if k is even [1]. We show that the prism over $K(n,k)$ and $B(n,k)$ is hamiltonian for $n > 2k$.

Another trend is the search for long cycles. For $K(n,k)$, the best lower bound currently known states the length of the longest cycle in O_k is $\sqrt{3|V(O_k)|}$, which is less than 3% for O_{10}, and asymptotically approaches zero as k increases. We improve this lower bound for O_k by providing a cycle with at least $.625|V(O_k)|$ vertices.

References

[1] L. R. Bueno and P. Horák. On hamiltonian cycles in the prism over the odd graphs. *Journal of Graph Theory*, 68(3):177–188, 2011.

[2] P. Horák, T. Kaiser, M. Rosenfeld, and Z. Ryjáček. The prism over the middle-levels graph is hamiltonian. *Order*, 22(1):73–81, 2005.

[3] T. Kaiser, Z. Ryjáček, D. Král, M. Rosenfeld, and H.-J. Voss. Hamilton cycles in prisms. *Journal of Graph Theory*, 56:249–269, 2007.

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Hamiltonian Cycles in 4-Connected 4-Regular Claw-free Graphs*

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Since the decision problem of the hamiltonian cycle problem is NP-Complete, one recent trend has been to search for long cycles or related structures. In this aspect, a hamiltonian prism is an interesting relaxation of a hamiltonian cycle [2]. The *prism over a graph* G is the Cartesian product $G\Box K_2$ of G with the complete graph on two vertices. A prism can be seen as the graph obtained by joining the corresponding vertices of two copies of G. A graph G is *prism-hamiltonian* if its prism has a hamiltonian cycle.

Plummer [3] has conjectured that every 4-connected 4-regular claw-free graph is hamiltonian and this conjecture remains open [1]. Also, the author has shown that 4-connected 4-regular claw-free graphs fall into three classes $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$, of which only $\mathcal{G}_1$ is known to be hamiltonian. In our work, we prove that $\mathcal{G}_0$ is hamiltonian and that $\mathcal{G}_2$ is prism-hamiltonian, also corroborating to a conjecture that the prism over every 4-connected 4-regular graph is hamiltonian [2].

Given a graph G, let $G^1 = G\Box K_2$ and $G^q = G^{q-1}\Box K_2$, for $q > 1$. We show that, for every connected graph G, it holds that G^q is hamiltonian for all $q \geq \lceil log_2\Delta(G)\rceil$, where $\Delta(G)$ is the maximum degree of G. Also, we show that this proof is equivalent to prove that $G\Box Q_n$ is prism-hamiltonian for some value of n where Q_n is the n-cube graph.

References

[1] H. J. Broersma, Zdenek Ryjácek, and Petr Vrána. How many conjectures can you stand? a survey. *Graphs and Combinatorics*, 28(1):57–75, 2012.

[2] T. Kaiser, Z. Ryjáček, D. Král, M. Rosenfeld, and H.-J. Voss. Hamilton cycles in prisms. *Journal of Graph Theory*, 56:249–269, 2007.

[3] M. D. Plummer. A note on Hamilton cycles in claw-free graphs. *Congressus Numerantium*, 96:113–122, 1993.

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Laplacian energy of special families of threshold graphs

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The concept of *Laplacian energy* of a graph G has been defined ([1], [2]) in 2006 as the sum of the absolute values of the differences between the eigenvalues of the Laplacian matrix and the average degree of the vertices of G. That is, if G is a connected graph with n vertices and m edges, the Laplacian energy of G is then

$$LE(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right| ,$$

where $\mu_1, \mu_2, \cdots, \mu_n$ is the sequence of Laplacian eigenvalues of G.

A *threshold* graph is a graph free of P_4, C_4 and $2K_2$.

In [3], imposing some restrictions on the spectra of a threshold graph, its Laplacian energy is computed. It is also proved that the pineapple with clique number $1 + \lfloor \frac{2n}{3} \rfloor$ has largest Laplacian energy among all the graphs satisfying those conditions. In this work, we construct two large families of threshold graphs, fixing the number of vertices and the clique number, that satisfy those restrictions. In this way, we prove that the above pineapple is an extremal graph for these families, respecting the Laplacian energy. Thereby, we corroborate a conjecture established in [4], indicating that this pineapple has maximum Laplacian energy among all connected graphs of the same order.

Furthermore, we exhibit pairs of such graphs with the same Laplacian energy and different Laplacian spectra, known as *Laplacian equienergetic graphs.*

References

[1] I. Gutman, B. Zhou, Laplacian energy of a graph, Lin. Algebra Appl., 414 (2006) 29-37.

[2] B. Zhou, I. Gutman, On Laplacian energy of graphs, MATCH Commun. Math. Comput. Chem. 57 (2007) 211-220.

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Adjacent strong edge-coloring of split-indifference graphs*

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Let G be a simple graph. An *adjacent strong edge-coloring* of G is a proper edge-coloring of G such that for each pair of adjacent vertices u, v of G, the set of colors assigned to the edges incident with u differs from the set of colors assigned to the edges incident with v. The *adjacent strong chromatic index*, denoted $\chi'_a(G)$, of G is the minimum number of colors required to produce an adjacent strong edge-coloring for G. It is clear that $\chi'_a(G) \geq \chi'(G)$, since every adjacent strong edge-coloring is an edge-coloring. It was proved that if the degrees of adjacent vertices are distinct, then $\chi'_a(G) = \Delta(G)$, where $\Delta(G)$ is the maximum degree of G. However, if G contains at least two adjacent vertices with maximum degree, then $\chi'_a(G) \geq \Delta(G)+1$ [1]. A *total coloring* of G assigns a color to each vertex and to each edge so that colored elements have different colors when they are adjacent or incident. The *total chromatic number* of G, denoted $\chi_T(G)$, is the minimum number of colors in a total coloring of G. For regular graphs G, $\chi'_a(G)$ and $\chi_T(G)$ are strongly related. Indeed, if G is a regular graph with at least three vertices, then $\chi'_a(G) = \chi_T(G)$ when $\chi_T(G) = \Delta(G) + 1$ [2].

In the seminal article on the adjacent strong edge-coloring, Z. Zhang et al. conjectured that every simple connected graph G with at least three vertices and $G \not\cong C_5$ (a 5-cycle) has $\chi'_a(G) \leq \Delta(G) + 2$ [1]. This conjecture is open for arbitrary graphs, but it holds for some classes of graphs [1, 2].

In this work, we focus our attention on graphs that are both split and indifference, a set that contains non-regular graphs. We prove the conjecture for split-indifference graphs. Moreover, we determine the adjacent strong chromatic index for split-indifference graphs with a universal vertex. For a split-indifference graph G without universal vertices, we give conditions for its adjacent strong chromatic index to be $\Delta(G) + 1$ and we conjecture that $\chi'_a(G) = \Delta(G) + 2$, otherwise.

References

[1] Z. Zhang, L. Liu, and J. Wang. Adjacent strong edge coloring of graphs. *Applied Mathematics Letters*, 15(5):623–626, 2002.

[2] Z. Zhang, D. R. Woodall, B. Yao, J. Li, X. Chen, and L. Bian. Adjacent strong edge colorings and total colorings of regular graphs. *Science in China Series A: Mathematics*, 52(5):973–980, 2009.

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Maximal independent sets in cylindrical grid graphs

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Finbow, Hartnell, and Whitehead [1] define, for every $t \in \mathbb{N}$, the set $\mathcal{M}_t$ as the set of graphs that have maximal independent sets of exactly t different sizes. The cylindrical grid graph is formed by the Cartesian product of the graph P_n, the path of length n, $n \geq 2$ and the graph C_m, the cycle of length m, $m \geq 3$, denoted by $P_n \square C_m$. Nandi, Parui, and Adhikari [2] propose methods to find the domination number in cylindrical grid graphs $P_n \square C_m$ with $m \geq 3$ and $n \in \{2, 3, 4\}$, and presented bounds on the domination numbers when $n = 5$ and $m \geq 3$. We present a method to find different sizes of maximal independent sets in a cylindrical grid graph and a lower bound for t, such that cylindrical grids belong to $\mathcal{M}_t$.

References

[1] A. Finbow, B. Hartnell, and C. Whitehead, A characterization of graphs of girth eight or more with exactly two sizes of maximal independent sets, *Discrete Math.* **125** (1994), 153-167.

[2] M. Nandi, S. Parui and A. Adhikari, The domination numbers of cylindrical grid graphs *Applied Math. and Comp.*, **217**, 10, (2011), 4879-4889.

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Editing to Cliques: A Survey of FPT Results and Recent Applications in Analyzing Large Datasets

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The talk will survey the parameterized complexity of CLUSTER EDIT, the problem of combinatorially editing a graph so that the result is a graph consisting of disjoint cliques. The problem comes in several flavours, including: (1) where the input is an ordinary undirected graph, and the information is certain about which pairs of vertices are related (edge) and which are not (non-edge), and (2) where the input is uncertain (edge, non-edge, or "maybe"). The latter has important applications in machine learning. The talk will also survey some recent implementations of FPT algorithms for CLUSTER EDIT and results analysing large data-sets in Ecology and Medicine.

On the generalized Helly property of hypergraphs and maximal cliques and bicliques*

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A family of sets is (p, q)-*intersecting* if every nonempty subfamily of p or fewer edges has at least q elements in its total intersection. A family of sets has the (p, q)-*Helly property* if every nonempty (p, q)-intersecting subfamily has total intersection of cardinality at least q. The $(2, 1)$-Helly property is the usual *Helly property*. A hypergraph is (p, q)-*Helly* if its edge family has the (p, q)-Helly property and *hereditary* (p, q)-*Helly* if each of its subhypergraphs has the (p, q)-Helly property. A graph is (p, q)-*clique-Helly* if the family of its maximal cliques has the (p, q)-Helly property and *hereditary* (p, q)-*clique-Helly* if each of its induced subgraphs is (p, q)-clique-Helly. The classes of (p, q)-*biclique-Helly* and *hereditary* (p, q)-*biclique-Helly graphs* are defined analogously, where a *biclique* is any set of vertices inducing a (possibly edgeless) complete bipartite graph. It is important to mention that our classes of $(2, 1)$-biclique-Helly graphs and hereditary $(2, 1)$-biclique-Helly graphs are different from the classes of 'biclique-Helly' and 'hereditary biclique-Helly' graphs defined by Groshaus and Szwarcfiter (Graphs Combin. 23 (2007) 633–645; Discrete Math. Theor. Comput. Sci. 10 (2008) 71–78) precisely because they do not regard stable sets as bicliques.

In this work, we prove several characterizations of hereditary (p, q)-Helly hypergraphs, including one by minimal forbidden partial subhypergraphs. On the algorithmic side, we give an improved time bound for the recognition of (p, q)-Helly hypergraphs and show that the recognition of hereditary (p, q)-Helly hypergraphs can be solved in polynomial time if p is fixed but it is co-NP-complete if p is part of the input. In addition, we generalize the characterization of p-clique-Helly graphs, in terms of expansions, to (p, q)-clique-Helly graphs and give different characterizations of hereditary (p, q)-clique-Helly graphs, including one by forbidden induced subgraphs. We give an improvement on the time bound for the recognition of (p, q)-clique-Helly graphs and prove that the recognition problem of hereditary (p, q)-clique-Helly graphs is polynomial-time solvable for p and q fixed but NP-hard if p or q is part of the input. Finally, we give different characterizations, recognition algorithms, and hardness results for (p, q)-biclique-Helly graphs and hereditary (p, q)-biclique-Helly graphs, analogous to those for (p, q)-clique-Helly and hereditary (p, q)-clique-Helly graphs.

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The Clique Problem parameterized by the degeneracy of a graph

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We show that the maximum clique of a graph G can be found in time $O((n - d)f(d))$ where n is the number of vertices, d is the degeneracy of G and $f(d)$ is the time for computing the maximum clique of a d–vertex subgraph of G. In other words, we show that the parameterization of the clique problem in terms of the degeneracy of a graph results in a fixed parameter tractable problem.

The *degeneracy* of a (simple) graph G is the minimum integer d such that every subgraph of G has a vertex of degree at most d. The degeneracy of a graph can be thought as a "measure of the sparcity" of the graph and can be computed in time $O(m + n)$ where m and n are the number of edges and vertices of the graph.

Our result is a simple extension of a result found in [1] where the authors show how to modify Bron–Kerbosch algorithm in order to guarantee its execution in time $O(nd3^{d/3})$ for a n–vertex graph G of degeneracy d.

We also show that some of the best performing algorithms reported in the literature for the maximum clique problem can be easily modified (or, in some cases, need no modification at all) in order to take advantage of this result.

References

[1] David Eppstein, Maarten Löffler, and Darren Strash. Listing all maximal cliques in sparse graphs in near-optimal time, June 2010.

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Vizinhança Mínima no Hipercubo

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Uma árvore Hamming-Huffman [1] é uma estrutura que une compressão de dados com detecção de erros. Esta árvore é uma extensão das árvores de Huffman onde cada nó de codificação possui um conjunto de nós, denominados nós de erros, responsáveis pela detecção de erros desta codificação. Cada uma dessas folhas de erro corresponde a uma codificação que possui distância Hamming 1 para uma folha de símbolo, assim, proibindo a utilização do prefixo que ela representa e garantindo a detecção de erros de 1 bit na mensagem compactada.

Na criação de uma árvore de Huffman [3] o principal problema a ser resolvido é a associação dos símbolos com as codificações mais adequadas às suas frequências, de acordo com o critério utilizado em um código de tamanho variável. Quando se trabalha com árvores Hamming Huffman é necessário considerar uma característica adicional, a quantidade de folhas de erros produzidas pelas folhas de símbolo. O custo da árvore Hamming Huffman parece estar relacionado com a quantidade de nós de erros produzidos no processo de construção desta árvore. Minimizar esta quantidade para um conjunto de codificações de um dado nível da árvore se traduz em achar a vizinhança mínima de um conjunto de nós em Q_b.

Neste trabalho, abordamos este problema utilizando [2] o grafo $Q2_b$. Este grafo é obtido a partir de Q_b através da seguinte operação: $Q_b^2 - Q_b$, ou seja, em $Q2_b$ dois nós compartilham uma aresta se a distância Hamming das codificações as quais eles representam for igual a dois. Desta maneira, o grafo $Q2_b$ possui dois componentes, um composto por codificações pares e outro por codificações ímpares. Para a solução deste problema estamos interessados em particular nas cliques de $Q2_b$. Ao longo do trabalho definimos diversas propriedades para estas cliques, assim como sua relação com a vizinhança em Q_b.

References

[1] R. W. Hamming. *Coding And Information Theory.* 1980.

[2] F. Harary and M. Livingston. Independent domination in hypercubes. *Applied Mathematics Letters*, 6:27–28, 1993.

[3] D. A. Huffman. A method of minimum redundancy codes. *Proceedings of the IRE*, pages 1098–1111, 1951.

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Enumeration of chordless cycles

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Enumeration is a fundamental task in computer science and many algorithms have been proposed for enumerating graph structures such as cycles, circuits, paths, trees and cliques. Due to the problem's instance sizes – which can be exponentially large – these kind of tasks are usually hard to deal with, since even small graph can have a huge number of such structures. However, in many practical problems enumeration is necessary. For example, the cycle enumeration is useful for analysis of Web and social networks and the number of cycles can be used to identify connectivity patterns in a network.

A structure that have received special attention is the chordless cycle, that is a cycle which is an induced subgraph. Since the vertex set of such cycle does not include the vertex set of any other cycle, it is considered *minimal*. Therefore, chordless cycles are good representatives of cyclic structures. Chordless cycles appear in connectivity structures of networks as a whole, ecological networks, such as food webs and in structure-property relationships in some chemical compound.

We developed two new algorithms, a sequential [1] and a parallel [2]. The sequential version was implemented in C++. The parallel version was implemented in OpenCL. In this talk, we present an algorithm to enumerate all chordless cycles of a given graph G, with an $\mathcal{O}(n \cdot P)$ complexity time, where P is the number of chordless paths in G and n is the number of vertices of G. The core idea of our algorithm also uses a vertex ordering scheme, in which any arbitrary cycle can be described in a unique way. With this in hand, we generate an initial set of vertex triplets and use a depth-first-search strategy to find all the chordless cycles.

References

[1] E.S. Dias, D. Castonguay, H. Longo, W.A.R. Jradi. *Efficient Enumeration of All Chordless Cycles in Graphs*, arXiv:1309.1051, September (2013).

[2] E.S. Dias, D. Castonguay, H. Longo, H.A.D. Nascimento, W.A.R. Jradi. *Fast Parallel Algorithm for Enumerating All Chordless Cycles in Graphs*, manuscript.

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Algoritmos certificadores e verificadores: testemunhas ausentes e provas computacionais*

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Um *algoritmo certificador* para um problema Π exibe, para uma instância x de Π, uma resposta y e uma *testemunha* w, possibilitando a verificação da corretude da resposta por meio de um *algoritmo verificador*, que recebe x, y e w como entrada. Algoritmos certificadores são em muitos casos preferíveis a algoritmos tradicionais (não-certificadores) porque permitem que acatemos as respostas obtidas como verdadeiras sem que precisemos confiar cegamente na *implementação* dos algoritmos que as encontraram, garantindo que as respostas não foram comprometidas por falhas na implementação.

Na literatura sobre algoritmos certificadores [2], busca-se em geral possibilitar uma verificação simples, de forma que a corretude do próprio verificador possa ser trivialmente comprovada, e eficiente, permitindo que a resposta seja verificada a partir da testemunha fornecida sem aumento significativo do tempo total de processamento. Há, no entanto, dois casos que fogem a esse padrão e que apresentam, ainda assim, interesse do ponto de vista de certificação/verificação. O primeiro caso é aquele em que conseguimos construir verificadores que prescindem de testemunhas, pois são capazes de efetuar a verificação de forma simples e eficiente diretamente da resposta obtida. O segundo é o caso em que a testemunha exibida permite uma verificação que não é formalmente eficiente, por demandar tempo exponencial, mas que, para instâncias pequenas, é computacionalmente viável, permitindo por exemplo a criação de provas computacionais para teoremas.

Ilustramos os dois casos acima, respectivamente, com algoritmos verificadores para o problema da seleção dos k maiores elementos [1] e o problema de reconhecimento de grafos de disco unitário [3].

References

[1] Blum, M., Floyd, R. W., Pratt, V. R., Rivest, R. L., Tarjan, R. E., Time bounds for selection, *Journal of Computer and System Sciences* 7 (4) (1973) 448–461.

[2] McConnell, R.M., Mehlhorn, S., Schweitzer, P., Certifying Algorithms, 2010. *Computer Science Review* 5(2) (2011) 119–161.

[3] McDiarmid, C., Müller, T., Integer realizations of disk and segment graphs, J. Comb. Theory, Series B 103(1) (2013) 114–143.

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Solving the k-in-a-tree problem for chordal graphs*

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The *three-in-a-tree algorithm*, proposed by Chudnovsky and Seymour [1], solves the following problem in polynomial time: given a graph with three prescribed vertices, check whether there is an induced tree containing these vertices.

In this work we deal with a generalization of this problem, known as *k-in-a-tree problem*. For the case where k is part of the input, the problem is known to be NP-complete [2]. For fixed k, the complexity of this problem is still open for $k \geq 4$. Polynomial time algorithms for the k-in-a-tree problem for $k \geq 4$ are known only for the cases of claw-free graphs, by Fiala et al. [3], and graphs with girth at least k, by Trotignon and Wei [4].

In this work we give a $\mathcal{O}(nm^2)$ time algorithm for the k-in-a-tree problem for chordal graphs, even in the case where k is part of the input.

References

[1] M. Chudnovsky and P. Seymour, The three-in-a-tree problem, Combinatorica 30 (2010), 387-417

[2] N. Derhy and C. Picouleau, Finding induced trees, Discrete Applied Mathematics 157 (2009) 3552-3557

[3] J. Fiala, M. Kamiński, B. Lidický and D. Paulusma, The k-in-a-path problem for claw-free graphs, Symposium on Theoretical Aspects of Computer Science (2010) 371-382

[4] N. Trotignon and L. Wei, The k-in-a-tree problem for graphs of girth at least k, Discrete Applied Mathematics 158 (2010) 1644-1649

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Minimal 4×4 M-obstruction cographs

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Let M be a symmetric $m \times m$ matrix over $0, 1, *$. An M-partition of a graph G is a partition of the vertex set $V(G)$ into m parts $V_1, V_2, \ldots, V_m$ such that: (i) V_i is a clique (respectively independent set) if $M(i,i) = 1$ (respectively $M(i,i) = 0$); (ii) there are all possible edges (respectively non-edges) between parts V_i and V_j, $i \neq j$, if $M(i,j) = 1$ (respectively $M(i,j) = 0$); (iii) there are no restrictions between parts V_i and V_j, $i \neq j$, if $M(i,j) = *$. A graph G that does not admit an M -partition is called an M -obstruction. A minimal M-obstruction is a graph G which is an M-obstruction, but such that every proper induced subgraph of G admits an M-partition. In [1] it is has been shown that matrix partition problems for cographs admit polynomial time algorithms and forbidden induced subgraph characterizations. Also, the authors bound the size of a largest minimal M-obstruction cograph.

This work provides explicit characterizations of M-partitionable cographs, in terms of minimal obstructions, for some 4x4 matrices M. More specifically, we have analyzed all matrices that correspond to partitions into 4 independent sets.

This work provides explicit characterizations of M-partitionable cographs, in terms of minimal obstructions, for some 4×4 matrices M. More specifically, we have analyzed all matrices that correspond to partitions into 4 independent sets.

References

[1] Generalized colourings (matrix partitions) of cographs. Feder, T. Hell, P., and Hochstättler W. Graph Theory in Paris: Proceedings of a Conference in Memory of Claude Berge - Page 149–168.

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Remarks on Complementary Prisms

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Complementary prism is a special case of a more general *complementary product* and a variation of the well-known *prism*. For a graph G with vertex set $V(G) = \{v_1, \ldots, v_n\}$ and edge set $E(G)$, the *complementary prism of* G is the graph denoted by $G\bar{G}$ with vertex set $V(G\bar{G}) = \{v_1, \ldots, v_n\} \cup \{\bar{v}_1, \ldots, \bar{v}_n\}$ and edge set $E(G\bar{G}) = E(G) \cup \{\bar{v}_i\bar{v}_j : 1 \leq i < j \leq n \text{ and } v_iv_j \notin E(G)\} \cup \{v_1\bar{v}_1, \ldots, v_n\bar{v}_n\}$.

In other words, the complementary prism $G\bar{G}$ of G arises from the disjoint union of the graph G and its complement $\bar{G}$ by adding the edges of a perfect matching joining pairs of corresponding vertices of G and $\bar{G}$. For every vertex u of G, we will consistently denote the corresponding vertex of $\bar{G}$ by $\bar{u}$. Similarly, if U is a set of vertices of G, then let $\bar{U} = \{\bar{u} : u \in U\}$ denote the corresponding set of vertices of $\bar{G}$. Let $V(G\bar{G}) = V(G) \cup V(\bar{G})$ where $V(\bar{G}) = \{\bar{v}_1, \ldots, \bar{v}_n\}$.

We study algorithmic/complexity properties of complementary prisms with respect to cliques, independent sets, k-domination, and especially P_3-convexity. We establish hardness results and identify some efficiently solvable cases. The description of our results is presented bellow.

Theorem 1 *Let d be a positive integer.*

For each of the following three properties, it is NP-complete to decide whether a given pair (G, k) where G is a graph and k is an integer has the property.

(i) $G\bar{G}$ has a clique of order k.

(ii) $G\bar{G}$ has an independent set of order k.

(iii) $G\bar{G}$ has a d-dominating set, that is, a set D of vertices of $G\bar{G}$ such that every vertex u in $V(G\bar{G}) \setminus D$ has at least d neighbors in D.

Theorem 2 *Let G be a graph.*

(i) If G has k components with $k \geq 2$, then $G\bar{G}$ has a P_3-hull set of order $k + 1$.

(ii) If G and $\bar{G}$ are connected, then $G\bar{G}$ has a P_3-hull set of order 5.

Theorem 3 *It is NP-complete to decide for a given pair (G, k) where G is a graph and k is an integer whether $G\bar{G}$ has a P_3-Carathéodory set of order k. Furthermore, for trees or cographs the minimum P_3-Carathéodory set can be determined in polynomial time.*

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Diameter of a Symmetric Icosahedral Fullerene Graph*

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A *fullerene* graph $G = (V, E)$ is planar, cubic and 3-connected with only pentagonal and hexagonal faces. All fullerene graphs have exactly 12 pentagonal faces. Recent studies indicate that fullerene graphs can contribute to the considerable development of medicine, chemistry, physics and engineering. Hence, its theoretical and applied importance.

Let $G = (V, E)$ be a connected graph. The *distance* $d(u, v)$ between two vertices $u, v \in V$, is the number of edges in a shortest path between u and v. The *diameter* $diam(G) = max\,\{d(u, v) : \; u, v \in V\}$ of a connected graph G is the biggest distance between two vertices of G.

We investigate the symmetry of these graphs based on the location of its pentagonal faces. More precisely, we say that a fullerene graph has *icosahedral symmetry* when the geometric centers of its 12 pentagonal faces give rise to an icosahedron. In this case, we just join the centers of the nearest pentagons, i.e., each center pentagon will match exactly 5 other centers. In a fullerene graph, each vertex belonging to a pentagonal face is said *pentagonal* vertex, otherwise it is called *hexagonal* vertex.

In 1937, Goldberg[2] suggested the use of a hexagonal lattice to represent polyhedra in the plane. Given two positive integer parameters i, j, Goldberg showed how to yield a *icosaedral fullerene* graph $G_{i,j}$. We will use this technique, devised by Goldberg, to represent the icosahedra in the plan generated from symmetric icosahedral fullerenes graphs. Andova and Skrekovski[1] solve the problem of the diameter of $G_{i,i}$ and $G_{i,0}$, finding $diam(G_{i,j}) = 6j + 4i - 1$ and conjectured that the diameter of a fullerene graph G on n vertices satisfies $diam(G) \geq \left\lfloor \sqrt{\frac{5}{3}n} \right\rfloor - 1$. Given a pair of positive integers $i > 0$ and $j \geq 2i + 1$, in this paper we establish the diameter $diam(G_{i,j}) = 6j + 2i - 1$.

References

[1] V. Andova,R. Skrekovski, *Diameter of fullerene graphs with full icosahedral symmetriy.*

[2] M. Goldberg, *A class of multi-symmetric polyhedra*, Tohoku Math. J. 43 (1939) 104-108.

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New results on the geodeticity of the contour of a graph*,†

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We consider only finite, simple and connected graphs G. Given a set $S \subseteq V(G)$, we say that the closed interval $I[S]$ of S is the set of vertices lying on shortest paths between any pair of vertices of S. The set S is *geodetic* if $I[S] = V(G)$. The *eccentricity* of a vertex v is the number of edges in the maximum shortest path between v and any vertex w of G. A vertex v is a *contour vertex* if no neighbor of v has an eccentricity greater than v. The *contour* $Ct(G)$ of G is the set formed by all contour vertices of G. A vertex w is an *eccentric vertex* of some vertex v if the distance between v and w is equal to the eccentricity of v. We denote $I^2[S] = I[I[S]]$.

In this work, we present some structural and computational results for two problems proposed by Cáceres et al. in 2005. The first of them is the problem of determining whether the contour of a graph is geodetic. The authors showed that the contour of distance-hereditary graphs is geodetic and there exist graphs with non-geodetic contour. The second one is the problem of deciding if there exists a graph G such that $I^2[Ct(G)] \neq V(G)$. This problem remains open until nowadays.

We prove that for any set of vertices S, if $Ct(G) \subseteq S$ and $|S| \geq |V(G)| - 3$, then S is a geodetic set. Every graph G presented in the literature whose $Ct(G)$ is non-geodetic is such that $|V(G) \setminus I[Ct(G)]| = 1$. Thus, our result implies that $I[Ct(G)]$ is geodetic for all of them. We present three infinite families of graphs whose contour is non-geodetic, particularly, one of them is such that $|V(G) \setminus I[Ct(G)]| > 3$. We also prove that for integers (a, b, c, d), $a \geq 3$, $b \geq 1$, $c \geq 1$, $d \geq 1$, there exists a graph with a contour vertices, b vertices that does not belong to $I[Ct(G)]$ and c contour vertices with d eccentric vertices which are not contour vertices. Finally, using computational tools we verified that if $|V(G)| < 10$, then $Ct(G)$ is geodetic; and there exist 4 non-isomorphic graphs with 10 vertices whose contour is non-geodetic and we present these graphs. As a corollary, if there exists a graph G such that $I^2[Ct(G)] \neq V(G)$, then $|V(G)| > 10$.

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O NÚMERO DE HELLY GEODÉTICO EM CONVEXIDADES*

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Dado um grafo G, um conjunto de vértices S de G é geodeticamente convexo se todos os vértices de qualquer caminho mínimo entre dois vértices de S pertencem a S.

Existem alguns parâmetros bastante estudados associados a convexidades em grafos, como posto, número de Radon e número de Carathéodory. Neste trabalho estudamos, na convexidade geodética, o parâmetro conhecido como número de Helly. Tal parâmetro é definido como o menor número inteiro k para o qual toda família $\mathcal{C}$, composta por conjuntos formados pelos vértices de G geodeticamente convexos e k-intersectante, possui um vértice comum a todos os conjuntos de $\mathcal{C}$. Denotamos o parâmetro por $h(G)$.

Determinamos o número de Helly para algumas classes de grafos, como as árvores, ciclos, grafos k-partidos completos, grafos distância hereditária e grades completas de dimensão d. Em todos os casos, exceto ciclos, o número de Helly é igual ao tamanho da clique máxima do grafo. Para ciclos de tamanho l, em que $l \neq 4$, o número de Helly é igual a três, já para ciclos de tamanho igual a quatro, temos que $h(C_4)$ é igual a dois. Mostramos também que somente para grafos completos K_n o número de Helly é igual a n.

Apresentamos uma caracterização parcial dos grafos que possuem número de Helly igual a dois e também um teorema cuja aplicação possibilita a determinação do parâmetro para certos grafos cordais e também para alguns grafos específicos. Tal teorema se vale da característica de alguns vértices simpliciais específicos no grafo G, que chamamos de restritos. Um vértice simplicial a em um grafo G é dito restrito quando sua vizinhança aberta induz uma clique não maximal em $G \setminus \{a\}$. Nessas condições, mostramos que o número de Helly do grafo G é igual ao do grafo $G \setminus \{a\}$.

Finalmente, são descritos dois limitantes inferiores para o número de Helly geodético de um grafo.

Keywords: convexidade, convexidade geodética, número de Helly.

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The problem of recognizing unit PI graphs

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Interval graphs, the intersection graphs of intervals on the real line, form a well-known and widely studied graph class since the sixties. Since then, many other graph classes have gained attention at least (if not mostly) due to their strict relation to the class of interval graphs. Corneil and Kamula [2] introduced the notion of point-interval graphs, or PI graphs, which is the intersection graphs of a family of triangles ABC between parallel lines L_1 and L_2 such that A lies on L_1 and $\overline{BC}$ is a segment of L_2. Many recognition approaches exist for interval graphs, Mertzios [3] provided an efficient algorithm for recognizing whether a given graph is a PI graph. Balof and Bogart [1] presented the notion of free triangle graphs, a generalization of PI graphs, which consists of the intersection graphs of a family of triangles ABC between parallel lines L_1 and L_2 such that A lies on L_1, B lies on L_2, and the location of C is free. If such an intersection model of triangles also satisfies the property of having all triangles with a unitary area, then the graph is called unit free triangle graph. The recognition of free triangle graphs is open, even in its unit case.

Aiming to tackle this latter dificulty, we pose the problem of recognizing the class of unit PI graphs. As partial results, we show that the class of unit PI graphs is a proper subset of the intersection of those of PI graphs (efficiently recognizable) and unit free triangle graphs (whose recognition problem is open). It is also shown that, contrary to PI graphs, unit PI graphs do not generalize interval graphs.

Besides, we show that the class of threshold signed graphs is included in a larger class that can be defined in terms of more graph-theoretical terms. A graph is threshold signed if there are positive real numbers S, T (the thresholds) and, for every vertex v, there is a real weight $w_v \leq \min\{S, T\}$ such that vu is an edge if and only if $|w_v + w_u| \geq S$ or $|w_v - w_u| \geq T$.

References:

[1] B. A. Balof and K. P. Bogart. Non-unit free triangle orders. *Order*, pages 21–35, 2006.

[2] D. G. Corneil and P. A. Kamula. Extensions of permutation and interval graphs. *Congressus Numerantium*, 58:267–275, 1987.

[3] G. B. Mertzios. The recognition of simple-triangle graphs and of linear-interval orders is easy. *CoRR*, abs/1210.4352, 2012.

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