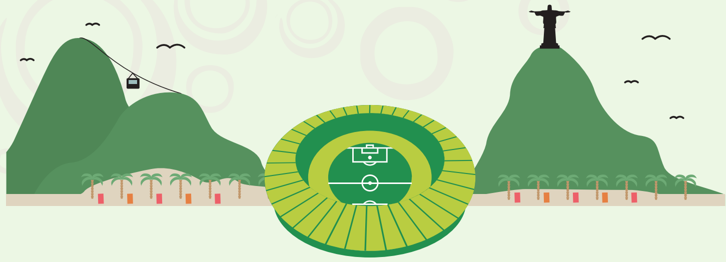
LAWCG 2018

VIII Latin American Workshop on Cliques in Graphs



ICM 2018 Satellite Event

August 9-11, 2018 Rio de Janeiro, RJ

PROGRAM and ABSTRACTS

Institutions and Societies

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IFORS International Federation of Operational Research Societies · ALIO · SOBRAPO · SBC · SBM Sociedade Brasileira de Matemática · impa · INMETRO

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VIII LATIN AMERICAN WORKSHOP ON CLIQUES IN GRAPHS

Federal Center of Technological Education of Rio de Janeiro (CEFET/RJ)
August 9-11, 2018
Rio de Janeiro, Brazil

We feel very honored to have been offered the responsibility to host the Latin American Workshop on Cliques in Graphs as a satellite event of the International Congress of Mathematicians. The first Latin American Workshop on Cliques in Graphs was held in Rio de Janeiro in 2002; the series continued with La Plata/Argentina (2006), Guanajuato/Mexico (2008), Itaipava/Brazil (2010), Buenos Aires/Argentina (2012), Pirenópolis/Brazil (2014), and La Plata/Argentina (2016).

Throughout all these years, the number of participants and lectures have increased significantly. In its eighth edition, the workshop reached the exceptional number of 80 contributed talks, most of them coauthored by students from all over the world: Argentina, Australia, Brazil, Canada, Chile, the United States, France, Mexico, Norway, and Switzerland. We are really grateful to all the participants for their contributions, in particular to the four invited speakers who generously accepted our invitation.

The aim of the Latin American Workshop on Cliques in Graphs is to promote a meeting of researchers in Graph Theory, Algorithms and Combinatorics, specially those working in Graph Operators, Intersection Graphs, Perfect Graphs, and related topics. The main goal in this series of workshops is to strengthen existing collaboration and to promote the creation of new international research groups.

We are grateful to the members of the Steering and Program Committees and especially to the members of the Organizing Committee at the Federal Center of Technological Education of Rio de Janeiro, the host institution, all members have worked hard in carrying out the many tasks necessary for successfully holding this meeting. We thank the significant financial support given by the Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ), the state research agency, through the Cientista do Nosso Estado project.

We are grateful to professors Jayme Luiz Szwarcfiter and Nelson Maculan Filho, who gave the wonderful opportunity for our workshop to take place as a satellite, occurring immediately after ICM 2018, so that the workshop can benefit from this unique moment for Mathematics in Brazil. The Latin American community of researchers in Combinatorics acknowledge the mirabilis year july 42 – july 43, when our professors Jayme and Maculan were born, with our thankful wishes and warmest regards to them!

Celina de Figueiredo, UFRJ
Cláudia Linhares Sales, UFC
Raphael Machado, INMETRO e CEFET
Rosiane de Freitas, UFAM

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Conference Program

August 9th

Registration at CEFET (09:00 - 10:00)

Room 1 (10:00 - 11:00)

1 Critical ideals of graphs and applications. *Carlos A. Alfaro, Jephian C.-H. Lin.*

2 Some Spectral Properties of Fulleroids-(3, 4, 6). *Celso M. da Silva Jr., Diego de S. Nicodemos.*

3 Some spectral properties of spider graphs. *Renata R. Del-Vecchio, Lucas L. S. Portugal, Celso M. da Silva Jr..*

4 The Zero-Divisor Graphs of the Direct Product of Commutative Rings. *André Ebling Brondani, Francisca Andrea Macedo França, Daniel Felisberto Traciná Filho.*

Room 2 (10:00 - 11:00)

5 Arc-disjoint branching flows. *A. Karolinna Maia, Jonas Costa, Raul Lopes.*

6 A linear algorithm to find the distance in Cayley Graph $H_{l,p}$. *C. S. R. Patrão, D. Castonguay, A. C. Ribeiro, L. A. B. Kowada.*

7 A pseudo-polynomial algorithm for the two-dimensional guillotine cutting stock. *Uéverton Souza, Leonardo Perazzini, Pedro Henrique González.*

8 Tessellations on graphs with few P_4's. *Alexandre Abreu, Franklin Marquezino, Daniel Posner.*

Coffee break (11:00 - 11:30)

Plenary Talk (11:30 - 12:30)

On the genus of dense graphs (Bojan Mohar).

Lunch (12:30 - 14:00)

Room 1 (14:00 - 15:00)

9 Pebbling in Semi-2-Trees. *Liliana Alcón, Marisa Gutierrez, Glenn Hurlbert.*

10 Determining optimum tree t-spanners for split graphs and cographs. *Fernanda Couto, Luís Cunha, Diego Ferraz.*

11 A kernelization algorithm for Closest String parameterized by the number of input strings. *M. B. Stockinger, U. S. Souza.*

12 Prismas complementares com 2-atribuição de papéis. *Diane Castonguay, Elisângela Silva Dias, Fernanda Neiva Mesquita.*

Room 2 (14:00 - 15:00)

13 A forbidden subgraph characterization of nested and 2-nested graphs. *Guillermo Durán, Luciano N. Grippo, Nina Pardal, Martín D. Safe.*

14 On Gallai and anti-Gallai chordal graphs. *G. A. Durán, F. Fernández Slezak, L. N. Grippo, F. S. Oliveira, M. D. Safe.*

15 Characterizing Star Graphs. *Guilherme de C. M. Gomes, Carlos V. G. C. Lima, Vinícius F. dos Santos.*

16 New results on dually-CPT graphs. *L. Alcón, N. Gudiño, M. Gutierrez.*

Room 1 (15:00 - 16:00)

17 Vector Domination Problem on the family of Split-Indifference Graphs. *Rodrigo Lamblet Mafort, Fábio Protti.*

18 Determinant families of dually chordal graphs. *Pablo De Caria.*

19 Characterization by forbidden subgraphs of near-bipartite P_4-tidy graphs. *Fábio Silva, Raquel Bravo, Rodolfo Oliveira, Uéverton Souza.*

20 Characterizing General Fullerene Graphs. *S. Dantas, L. Faria, A. Furtado, S. Klein, D. Nicodemos.*

Room 2 (15:00 - 16:00)

21 On the adjacent vertex-distinguishing total coloring of power of cycles. *J.D. Alvarado, S. Dantas.*

22 Identifying codes in complementary prisms. *Juliana Paula Félix, Márcia Rodrigues Cappelle.*

23 On identifying codes in the Cartesian product of a star and a path. *Juliana Paula Félix, Márcia Rodrigues Cappelle.*

24 Sobre Códigos Corretores de Distância Hamming 3. *Natália Pedroza, Paulo E. Pinto, Jayme L. Szwarcfiter.*

Coffee break (16:00 - 16:30)

Room 1 (16:30 - 17:30)

25 The Biclique Graph of K3-free Graphs are the Square of Some Graph. *Marina Groshaus, André L. P. Guedes.*

26 On the Diameter of Spherical Fullerene Graphs. *S. Dantas, V. Linder, D. Nicodemos.*

27 Some forbidden structures for the near-bipartition problem on distance-hereditary graphs. *Rodolfo Oliveira, Raquel Bravo, Uéverton Souza, Fabio Silva.*

28 Caracterização estrutural de grafos-(1, 2) bem-cobertos. *S. R. Alves, F. Couto, L. Faria, S. Gravier, S. Klein, U. dos S. Souza.*

Room 2 (16:30 - 17:30)

29 An optimal algorithm to totally color some power of cycle graphs. *Alesom Zorzi, Celina de Figueiredo, Raphael Machado.*

30 Equitable total coloring of graphs with universal vertex. *Mayara Midori Omai, Sheila Morais de Almeida, Diana Sasaki Nobrega.*

31 A Recolouring Procedure for Total Colouring. *L. M. Zatesko, R. Carmo, A. L. P. Guedes.*

32 The b-continuity of graphs with large girth. *Allen Ibiapina, Ana Silva.*

Welcome Ceremony at UERJ (17:30)

August 10th

Room 1 (9:00 - 10:00)

33 On the null structure of bipartite graphs without cycles of length multiple of 4. *Daniel A. Jaume, Gonzalo Molina, Adrián Pastine.*

34 Proper gap-labellings of unicyclic graphs. *C. A. Weffort-Santos, C. N. Campos, R. C. S. Schouery.*

35 A Decomposition for Edge-colouring. *João Pedro W. Bernardi, Sheila M. de Almeida, Leandro M. Zatesko.*

36 The Colourability problem on (r, l)-Graphs and a few parametrized solutions. *M. S. D. Alves, U. S. Souza.*

Room 2: (9:00 - 10:00)

37 Colorings, Cliques and Relaxations of Planarity. *Val Pinciu.*

38 Algoritmos para os Casos Polinomiais da Coloração Orientada. *Mateus de Paula Ferreira, Hebert Coelho da Silva.*

39 Equitable total coloring of classes of tripartite complete graphs. *A.G. da Silva, D. Sasaki, S. Dantas.*

40 Graphs with small fall-spectrum. *Ana Silva.*

Room 1 (10:00 - 11:00)

41 Coloring Game: characterization of a $(3, 4^*)$-caterpillar. *S. Dantas, C.M.H. de Figueiredo, A. Furtado, S. Gravier.*

42 Matching problem for vertex colored graphs. *Martín Matamala.*

43 Alguns Resultados em Coloração Orientada e Clique Coloração Orientada. *Hebert Coelho, Luerbio Faria, Sylvain Gravier, Sulamita Klein.*

44 On the convexity number for complementary prisms. *Diane Castonguay, Erika M. M. Coelho, Hebert Coelho, Julliano R. Nascimento.*

Room 2 (10:00 - 11:00)

45 Knot-Free Vertex Deletion Problem: Parameterized Complexity of a Deadlock Resolution Graph Problem. *Alan D. A. Carneiro, Fábio Protti, Uéverton S. Souza.*

46 A Parameterized Complexity Analysis of Clique and Independent Set in Complementary Prisms. *Priscila Camargo, Alan D. A. Carneiro, Uéverton S. Souza.*

47 The Diverse Vertex Covers Problem. *Julien Baste, Michael R. Fellows, Lars Jaffke, Mateus de Oliveira Oliveira, Frances A. Rosamond.*

48 Directed tree-width is FPT. *A. Karolinna Maia, Raul Lopes, Victor Campos.*

Coffee break (11:00 - 11:30)

Plenary Talk (11:30 - 12:30)

Clique Operators in Digraphs (Marisa Gutierrez).

Lunch (12:30 - 14:00)

Room 1 (14:00 - 15:00)

49 P_3-Helly number of graphs with few P_4. *Moisés T. Carvalho, Simone Dantas, Mitre C. Dourado, Daniel Posner, Jayme L. Szwarcfiter.*

50 On the P_3-Hull Number for Strongly Regular Graphs. *Erika M. M. Coelho, Braully R. Silva , Hebert Coelho.*

51 Sobre o número de Sierksma de um grafo. *Felipe Pereira do Carmo, Carlos Alberto de Jesus Marthinon, Uéverton dos Santos Souza, Moisés Teles Carvalho Junior.*

52 The Rank on the Graph Geodetic Convexity. *M.T. Carvalho, S. Dantas, C.V.G.C. Lima, V. Linder, V.F. dos Santos.*

Room 2 (14:00 - 15:00)

53 Covering a body using unequal spheres and the problem of finding covering holes. *Helder Manoel Venceslau, Marilis Bahr Karam Venceslau, Nélson Maculan.*

54 New proposals for the Problem of Covering Solids using Spheres of Different Radii. *Pedro Henrique González, Ana Flavia U. S. Macambira, Renan Vicente Pinto, Luidi Simonetti, Nelson Maculan, Philippe Michelon.*

55 Connectivity of cubical polytopes. *Hoa Bui Thi, Guillermo Pineda-Villavicencio, Julien Ugon.*

56 A strategy to select vertices as candidates for routers in a Steiner tree. *João Guilherme Martinez, Rosiane de Freitas, Altigran da Silva, Fábio Protti.*

Room 1 (15:00 - 16:00)

57 Powers of Circular-Arc Models. *Francisco J. Soulignac, Pablo Terlisky.*

58 Digrafo de intersecccíon de torneos transitivos maximales. *G. Sánchez Vallduví, M. Gutiérrez, B. Llano.*

59 Extremal unit circular-arc models. *Francisco J. Soulignac, Pablo Terlisky.*

60 Circular-arc Bigraphs and the Helly subclass. *Marina Groshaus, André Luiz Pires Guedes, Fabricio Schiavon Kolberg .*

Room 2 (15:00 - 16:00)

61 Pursuit Games on graphs with few P4's. *Nicolas Martins, Rudini Sampaio.*

62 Clobber game as executive functions test. *T. Pará, S. Dantas, S. Gravier, L.A.V. de Carvalho, P. Mattos.*

63 On the Minimum Broadcast Time Problem. *Diego Delle Donne, Ivo Koch.*

64 On distance colorings, graph embedding and IP/CP models. *Rosiane de Freitas, Bruno Dias, Nelson Maculan, Javier Marrenco, Philippe Michelon, Jayme Szwarcfiter.*

Coffee break (16:00 - 16:30)

Plenary Talk (16:30 - 17:30)

New results on intersecting families of subsets (Gyula O.H. Katona).

Social Event at CEFET (17:30)

August 11th

Room 1 (9:00 - 10:00)

65 On bicliques and the second clique graph of suspensions. *M.A. Pizaña, I.A. Robles.*

66 On a Class of Proper 2-Thin Graphs. *M. S. Sampaio Jr., F. S. Oliveira, J. L. Szwarcfiter.*

67 On Orthodox Tree Representations of $K_{n,m}$. *C.F. Bornstein, J.W. Coura Pinto, J.L. Szwarcfiter.*

68 Biclique Graphs of Interval Bigraphs and Circular-arc Bigraphs. *E. P. Cruz, M. Groshaus, A. L. P. Guedes.*

Room 2 (9:00 - 10:00)

69 B_1-EPG-Helly Graph Recognition. *Claudson Bornstein, Tanilson Santos, Uéverton Souza, Jayme Szwarcfiter.*

70 The Terminal connection problem on strongly chordal graphs and cographs. *A. A. Melo, C. M. H. Figueiredo, U. S. Souza.*

71 Complexity Analisys of the And/Or graph Solution Problem on Planar Graphs. *M. R. Alves, U. S. Souza.*

72 Weighted proper orientations of trees and graphs of bounded treewidth. *Julio Araujo, Cláudia Linhares Sales, Ignasi Sau, Ana Silva.*

Room 1 (10:00 - 11:00)

73 Sobre los grafos PVPG: una subclase de los grafos vértice intersección de caminos en una grilla. *Liliana Alcón, Flavia Bonomo, María Pía Mazzoleni, Fabiano Oliveira.*

74 Clique-divergence is not first-order expressible for the class of finite graphs. *Carmen Cedillo, Miguel Pizaña.*

75 On Clique-Inverse Graphs of Graphs with Bounded Clique Number. *Liliana Alcón, Sylvain Gravier, Claudia Sales, Fabio Protti, Gabriela Ravenna.*

76 The unit-demand envy-free princing problem applied to the sports entertainment industry. *Marcos Salvatierra, Rosiane de Freitas.*

Room 2 (10:00 - 11:00)

77 Worst cases in constrained LIFO pick-up and delivery problems. *Sebastián Urrutia, Dominique de Werra.*

78 Size Multipartite Ramsey Number. *Pablo Henrique Perondi, Emerson Luiz do Monte Carmelo.*

79 Maximum number of edges in graphs with prescribed maximum degree and matching number. *Pinar Heggernes, Jean R. S. Blair, Paloma T. Lima.*

80 An extremal problem on the interval counts. *L. S. Medeiros, F. S. Oliveira, J. L. Szwarcfiter.*

Coffee Break (11:00 - 11:30)

Plenary Talk (11:30 - 12:30)

Minimizing the Solid Angle Sum of Orthogonal Polyhedra and Edge Guarding (Jorge Urrutia).

Social Event at Lapa (19:00 - 22:00)

Plenary talks

On the genus of dense graphs

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Abstract

What is the smallest genus of a surface in which the complete graph K_n can be embedded? This question, known as the *Heawood problem*, was resolved in 1968 by Ringel and Youngs and its solution gave birth to topological graph theory. In the 1990s, Archdeacon and Grable and Rödl and Thomas proved that the genus of random graphs behaves very much like the genus of complete graphs.

The speaker will outline some recent results about genus embeddings of dense graphs building on the work outlined above. The work, which was originally motivated by algorithmic questions, uses modern notions of quasi-randomness and graph limits, and leads to interesting new problems in topological graph theory.

Substantial part of the talk will be based on recent joint work with Yifan Jing.

Clique Operators in Digraphs

Marisa Gutierrez
Departamento de Matemática Universidad Nacional de La Plata, Argentina. CONICET

Keywords: disimplex, dicliques, transitive tournaments

Doing a review of the results obtained on the clique operator, a lot of water has flowed under the bridge...

We know what graphs live in the image, how difficult is to decide about them. We have played with the iterated clique operator and obtained results on convergence, divergence, periodicity, etc. Recently we have learned that clique-convergence is undecidable for infinite graphs. We have analyzed its behavior in certain classes of graphs. And as if this were not enough, there were also experts who got involved with the bicliques of a graph. These results have seen the light mainly in Latin America and that is why we are here!

In this talk we will show you what currently has us trapped, new versions of the clique operator but in directed graphs, the following two new operators:

Transitive Tournament Operator τ

- V(τ(D)) is the set of maximal transitive subtournaments of D (maximal by inclusion).
- A(τ(D)): if T_1 and T_2 are vertices of D and f_1, f_2, s_1, s_2 the corresponding sources and sinks, then $T_1 \to T_2$ iff $s_1, f_2 \in T_1 \cap T_2$ and $f_1, s_2 \notin T_1 \cap T_2$

Diclique Operator $\overrightarrow{K}$

- $V(\overrightarrow{K}(D))$ is the set of dicliques D that are maximal disimplex of D.
- $A(\overrightarrow{K}(D))$: if (A, B) and (A', B') are dicliques of D, then $(A, B) \longrightarrow (A', B')$ iff $B \cap A' \neq \emptyset$.

We will present our first results about them, the convergency, divergency and the behavior on certain classes of digraphs.

New results on intersecting families of subsets

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Abstract

Let $[n] = \{1, 2, \ldots, n\}$ be the underlying set. A family $\mathcal{F} \subset 2^{[n]}$ of its subsets is called intersecting if $F, G \in \mathcal{F}$ implies $F \cap G \neq \emptyset$. It is trivial that the largest intersecting family has 2^{n-1} members. The situation is different when only k-element subsets are considered, that is $\mathcal{F} \subset \binom{[n]}{k}$. The celebrated Erdős-Ko-Rado theorem says that if $k \leq \frac{n}{2}$ then an intersecting family of k-element sets cannot have more than $\binom{n-1}{k-1}$ members. Equality can be obtained for the family of sets containing one fixed element.

We will survey some of the results of the history of the area and show some new developments. One such direction is the problem of "two-part intersecting" families. The underlying set $[n]$ is partitioned into X_1 and X_2. Is it still true that the largest intersecting family is the one consisting of members containing one fixed element? It is perhaps surprising that the answer is yes. Even in the following very general form. Some positive integers $k_i, \ell_i (1 \leq i \leq m)$ are given. We prove that if $\mathcal{F}$ is an intersecting family containing members F such that $|F \cap X_1| = k_i, |F \cap X_2| = \ell_i$ holds for one of the values $i (1 \leq i \leq m)$ then $|\mathcal{F}|$ cannot exceed the size of the largest subfamily containing one element. The statement was known for the case $m = 2$ as a result of Frankl.

The shadow $\sigma(\mathcal{F})$ is defined for the case when $\mathcal{F} \subset \binom{[n]}{k}$. It is a family of all $k-1$-element subsets of members of $\mathcal{F}$. The shadow theorem determines the minimum size of the shadow family for fixed n, k and $|\mathcal{F}|$. The optimal family consists of sets being at the "beginning" of $[n]$. This family often contains disjoint pairs. Therefore to find the smallest shadow of an intersecting family is very different from the traditional problem. We will introduce some old and new results of this kind.

Minimizing the Solid Angle Sum of Orthogonal Polyhedra and Edge Guarding

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Ciudad de México, México.

A well known problem in Art Galleries and illumination problems, is that any orthogonal polygon can always be guarded using $\lfloor \frac{n}{4} \rfloor$ guards. In this talk we extend these results to edge guarding of orthogonal polyhedron in $\mathbb{R}^3$.

To obtain our results, we generalize generalize to $\mathbb{R}^3$ the well-known result that in an orthogonal polygon with n vertices, $(n+4)/2$ of them are convex and $(n-4)/2$ of them are reflex. We define a vertex of a polyhedron to be convex on the faces if it is convex or straight in all the faces where it participates, and to be reflex on the faces otherwise. If a polyhedron with n vertices and genus g has k vertices of degree greater than 3 (in its 1-skeleton), we prove that it has $(n+8-8g+3k)/2$ vertices that are convex on the faces and $(n-8+8g-3k)/2$ vertices that are reflex on the faces.

We also give a characterization for the orthogonal polyhedron in $\mathbb{R}^3$ that minimize the sum of its internal solid angles, and prove that their minimum angle sum is $(n-4)\pi$ and their maximum angle sum is $(3n-24)\pi$.

If time allows, we will prove that if the orthogonal polyhedron has k_4 vertices of degree 4, k_6 vertices of degree 6, genus g and h_m holes on its faces, then we can guard it using at most $(11e-k_4-3k_6-12g-24h_m+12)/72$ $\frac{\pi}{2}$-edge guards (i.e., having a visibility angle of $\pi/2$ in the relative interior of each edge).

References

[1] J. Abello, V. Estivill-Castro, T. Shermer, and J. Urrutia: Illumination of orthogonal polygons with orthogonal floodlights. Internat. J. Comp. Geom. 8: 25–38 (1998).

[2] N.M. Benbernou, E.D. Demaine, M.L. Demaine, A. Kurdia J. O'Rourke, G. Toussaint, J. Urrutia, and G. Viglietta: Edge-guarding Orthogonal Polyhedra): CCCG 2011, Toronto ON, August 10-12, 2011, pp. 461–466.

[3] J. Urrutia: Art gallery and illumination problems. In J.- R. Sack and J. Urrutia, editors, Handbook of Computational Geometry, pages 973–1027. North-Holland, 2000.

[4] G. Viglietta: Face-Guarding Polyhedra. Computational Geometry: Theory and Applications, vol. 47, no. 8, pp. 833–846, 2014.

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Contributed talks

(The order number in the program is the page number)

Critical ideals of graphs and applications

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Keywords: critical ideals, minimum rank, zero forcing number.

Given a graph G and a set of indeterminates $X_G = \{x_u : u \in V(G)\}$, the *generalized Laplacian matrix* $L(G, X_G)$ of G is the matrix whose uv-entry is given by x_u, if $u = v$, and the number $-m_{uv}$ of edges between vertices u and v, otherwise. Let $\mathcal{R}[X_G]$ denote the polynomial ring over a commutative ring $\mathcal{R}$ with unity in the variables X_G, then for $1 \leq i \leq n$ the i-th *critical ideal* $I_i^{\mathcal{R}}(G, X_G)$ of G are the determinantal ideals spanned by $\langle \text{minors}_i(L(G, X_G)) \rangle \subseteq \mathcal{R}[X_G]$, where n is the number of vertices of G and $\text{minors}_i(L(G, X_G))$ is the set of the determinants of the $i \times i$ submatrices of $L(G, X_G)$.

Initially, critical ideals were defined as a generalization of the critical group, also known as sandpile group. Furthermore, the varieties associated to these ideals can be regarded as a generalization of the Laplacian and Adjacency spectra of G. Recently, there have been found relations between the zero forcing number and the minimum rank of a graph with the algebraic co-rank.

In this talk, we are going to outlook how all these concepts are related. And show few characterizations for these parameters where cliques and stable sets play an important role.

Some Spectral Properties of Fulleroids-$(3,4,6)$

Celso M. da Silva Jr. [1] Diego de S. Nicodemos [2,*]
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[2] Colégio Pedro II, Brazil.

Keywords: Fullerene Graphs, Fulleroid-$(3,4,6)$ *Graphs, Adjacency Matrix, Eigenvalue and Odd Cycle Transversal.*

Let $G = (V, E)$ be a simple, finite and undirected graph on n vertices. A set of edges of G is an *odd cycle (edge) transversal* if its removal results in a bipartite graph. According to [1] finding a minimum odd cycle transversal of a graph is equivalent to partitioning the vertex set into two parts, such that the number of edges between the two parts is maximum (max-cut problem). The *adjacency matrix* of G, $A(G) = [a_{i,j}]$, is the square matrix of order n, such that $a_{i,j} = 1$, if v_i and v_j are adjacent and $a_{i,j} = 0$, otherwise. An eigenvalue of a graph G is an eigenvalue of its adjacency matrix $A(G)$.

A *fullerene graph* is a 3-connected 3-regular planar graph with only pentagonal and hexagonal faces. Actually fullerene graphs model fullerene molecules and there is a great scientific interest in discovering/determining parameters of fullerenes graphs related to fullerene molecule stability. A number of invariants of these graphs have been examined recently as possible predictors of fullerene stability, including, for example, the smallest size of an odd cycle transversal and its smallest eigenvalue.

In this work we discuss a variation of a fullerene graph: a *fulleroid*-$(3,4,6)$ graph which is a cubic 3-connected planar graph with all faces of size 3, 4 or 6. In particular, we obtain an upper bound on its smallest eigenvalue by using the relationship between this paramater and the smallest size of an odd cycle transversal.

References

[1] Faria, L.; Klein, S. and Stehlík, M., *Odd Cycle Transversals and Independent Sets in Fullerene Graphs*, SIAM Journal of Discrete Mathematic Vol. 48 (2012), 1458–1469.

Some spectral properties of spider graphs

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Keywords: Spider graphs, Signless Laplacian matrix, Distance matrix, distance signless Laplacian matrix.

We call G a P_4-sparse graph [1] if every set of five vertices in G induces at most one P_4. This class of graphs extends the well-known class of cographs and its subclass of thereshold graphs, both with already known spectral properties. In [2], it was proved that G is a connected P_4-sparse graph with connected complement if and only if G is a spider whose head, if exists, induces a P_4-sparse graph.

Given a connected graph G, let $D(G)$ be the distance matrix of G, and let $T(G)$ be the diagonal matrix of the row sums of $D(G)$. In analogy to signless matrix of G, $Q(G)$, it was recently introduced the matrix $D^Q(G) = T(G) + D(G)$, called the distance signless Laplacian of G [3].

In this work we study some spectral properties of spider graphs, an important class to characterize P_4-sparse graphs. In particular, we discuss the M-spectrum of these graphs, in case where M is the signless Laplacian matrix, distance matrix and distance signless Laplacian matrix. Bounds for their M-eigenvalues are shown, including Nordhaus-Gaddum type relations.

References:

[1] Hoang, Trong Chinh. A class of perfect graphs. Diss. McGill University Libraries, 1983.
[2] Jamison, Beverly, and Stephan, Olariu. ”A tree representation for P4-sparse graphs.” Discrete Applied Mathematics 35.2 (1992): 115-129.
[3] Aouchiche, Mustapha, and Pierre Hansen. ”Two Laplacians for the distance matrix of a graph.” Linear Algebra and its Applications 439.1 (2013): 21-33.

The Zero-Divisor Graphs of the Direct Product of Commutative Rings

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Keywords: Graph. Commutative ring. Zero-divisor graph.

Abstract

Let R be a commutative ring with nonzero identity. The idea of a zero-divisor graph of R was introduced by Beck in [2], where he was mainly interested in colorings of R. Our definition of a zero-divisor graph of R, denoted by $\Gamma(R)$, and the emphasis on the interplay between the graph-theoretic properties of $\Gamma(R)$ and the ring-theoretic properties of R are due to Anderson and Livingston in [1]. Given a positive integer k, be $p_1, p_2, \ldots, p_k$ integers such that $p_\ell \leq p_{\ell+1}$, $1 \leq \ell \leq k-1$, and consider the ring $R' \simeq F_{p_1} \times F_{p_2} \times \ldots \times F_{p_k}$, where F_{p_i}, $1 \leq i \leq k$, is a field with p_i elements. In this work we will show some results, obtained during the study of the structure in $\Gamma(R')$. Among them, we determine an algebraic expression for the degree of each vertex of $\Gamma(R')$ as a function of the orders of each field, and, in particular, we obtain expressions for the maximum and minimum degrees of the graph. In addition, we find a condition necessary for a given graph to be a zero-divisor graph of a ring of the type R'.

References

[1] D. F. Anderson and P. S. Livingston. The zero-divisor graph of a commutative ring. *Journal of Algebra*, 217(2):434–447, 1999.

[2] I. Beck. Coloring of commutative rings. *Journal of Algebra*, 116:208–226, 1988.

Arc-disjoint branching flows[3]

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Keywords: branching flow, arc-disjoin flows, branchings

A network $\mathcal{N} = (V, A, u)$ is a digraph $D = (V, A)$ equipped with a capacity function $u : A \to \mathbb{Z}_+$. A flow in a network $\mathcal{N}$ is a function $x : A \to \mathbb{Z}_+$. We denote the value of x on the arc $vw \in A$ and the capacity of the same arc by x_{vw} and u_{vw}, respectively. The balance vector of a flow x is a function b_x which gives to each vertex $v \in V$ the value: $b_x(v) = \sum_{vw \in A} x_{vw} - \sum_{zv \in A} x_{zv}$. A flow x is feasible if $x_{vw} \leq u_{vw}$, for all $vw \in A$.

An s-branching flow on a network $\mathcal{N}$ with n vertices is a flow x with balance vector $b_x(s) = n - 1$ and $b_x(v) = -1$ for all $v \in V \setminus \{s\}$. We focus on the problem of finding multiple arc-disjoint branching flows on a given network, as introduced in [1]. We study its complexity on networks when considering fixed capacities on the arcs. A previous result from [2] shows that, as a consequence of the Exponential Time Hypothesis (ETH) [3], this problem is hard in networks on n vertices where all the arcs have capacity equal to $n - f(n)$, where f is an integer function such that $(\log(n))^{1+\epsilon} \leq f(n) \leq n/2$ for $\epsilon > 0$. We extended this result showing that, under the same assumptions, the problem is also hard when the capacities are equal to $f(n)$.

References

[1] J. Bang-Jensen and S. Bessy, "(Arc-) disjoint flows in networks," *Theoretical Computer Science*, vol. 526, pp. 28–40, 2014.

[2] J. Bang-Jensen, F. Havet, and A. Yeo, "The complexity of finding arc-disjoint branching flows," *Discrete Applied Mathematics*, vol. 209, pp. 16–26, 2016.

[3] R. Impagliazzo, R. Paturi, and F. Zane, "Which problems have strongly exponential complexity?," *Journal of Computer and System Sciences*, vol. 63, no. 4, pp. 512 – 530, 2001.

[2]Supported by CAPES.
[3]This work was partly supported by CNPq - Universal project 425297/2016-0 and FUNCAP - PRONEM PNE-0112-00061.01.00/16.

A linear algorithm to find the distance in Cayley Graph $H_{\ell,p}$

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Keywords: Interconnection networks; Grafo de Cayley; Distance

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The family of graphs $H_{\ell,p}$ has been defined in the context of edge partitions [1]. The established properties such as vertex-transitivity and low diameter suggest this family as a good topology for the design of interconnection networks. The $p^{\ell-1}$ vertices of the graph $H_{\ell,p}$ are the ℓ-tuples with values between 0 and $p-1$, such that the sum of the ℓ values is a multiple of p, and there is an edge between two vertices, if the two corresponding tuples have two pairs of entries whose values differ by one unit. The distance between two vertices in a graph is the number of edges in a shortest path connecting them.

In this work, as the diameter of the graph $H_{\ell,p}$ is $\theta(\ell \cdot p)$ [2], then any algorithm to show the path between two vertices needs $\Omega(\ell \cdot p)$ steps, however we show that to find its distance can be done in $\theta(\ell \cdot \log p)$, which is the input size in bits, and therefore our algorithm has optimal asymptotic complexity.

References

[1] Holyer, I.: The NP-Completeness of Some Edge-Partition Problems, *SIAM Journal on Computing* **10(4)** (1981) 713–717.

[2] Castonguay, D., Ribeiro, A. C., Kowada, L. A. B., Figueiredo C. M. H.: On the diameter of the Cayley Graph $H_{\ell,p}$, *Matemática Contemporânea* **44** (2015).

A pseudo-polynomial algorithm for the two-dimensional guillotine cutting stock

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Keywords: two-dimensional guillotine cutting stock, And-Or graphs, dynamic programming

Producing parts from a rectangular plate P is a common and important industrial problem. Whenever the strategy to extract the parts from P is a guillotine cut, this combinatorial optimization problem is known as two-dimensional guillotine cutting stock problem. This problem consists in determining a sequence of cuts to be made by a guillotine in P (denoted cutting pattern) in order to: generate a subset of parts; minimize waste of material; and consequently maximize the profit of production. Motivated by its relevance in the industry, this work combines the concepts of graphs And-Or and dynamic program to develop an algorithm to solve the problem in pseudo-polynomial time. The use of searches in And-OR graphs to produce a pseudo-polynomial algorithm, besides being a novel approach to the literature, it produces an algorithm in with it worst case complexity coincides with the state of the art to the problem.

Tessellations on graphs with few P_4's

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Keywords: graph tessellations, graphs with few P_4's, quasi-threshold graphs.

Given a graph G, a tessellation is a set of disjoint cliques that covers the vertex set $V(G)$. The tessellation cover number $T(G)$ represents the minimum number of tessellations needed such that their union covers the edge set $E(G)$. Abreu et al. [1] presented an upper bound for $T(G)$ showing that the chromatic number of the clique graph of G, $\chi(K(G))$, is a tight upper bound. Abreu et al. [2] improved this upper bound to $T(G) \leq min\{\chi(K(G)), \chi'(G)\}$, where $\chi'(G)$ is the chromatic index of G. They also defined $T(G)$ for restricted graphs classes and analyzed the complexity of the t-TESSELLABILI TY problem, proving that it is $\mathcal{NP}$-complete for $t \geq 3$, and is linear-time solvable when $t = 2$. In this work we show that some operations in graphs, such as the union operation and the addition of a true twin vertex, do not affect the tessellation cover number of the resulting graphs. Such results allowed us to prove bounds of the tessellation cover number and to establish efficient polynomial time algorithms for graph classes with few induced paths of size four, such as quasi-threshold graphs.

References

[1] A. Abreu, L. Cunha, T. Fernandes, C. de Figueiredo, L. Kowada, F. Marquezino, D. Posner, and R. Portugal. Bounds and complexity for the tessellation problem. *Mat. Contemp. (accepted)*, 2017.

[2] A. Abreu, L. Cunha, T. Fernandes, C. de Figueiredo, L. Kowada, F. Marquezino, D. Posner, and R. Portugal. The graph tessellation cover number: extremal bounds, efficient algorithms and hardness. *LATIN 2018 (accepted), preprint arXiv:1712.07476*, 2018.

Pebbling in Semi-2-Trees

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Keywords: pebbling number, 2-trees, 2-paths, complexity, NP*-complete*

The fundamental question in graph pebbling is whether a given supply (*configuration*) of discrete pebbles on the vertices of a connected graph can reach a particular *root* vertex r. The operation of pebble movement across an edge $\{u, v\}$ is called a *pebbling step*: while two pebbles cross the edge, only one arrives at the opposite end, as the other is consumed. A configuration that can reach r is *r-solvable*.

The *size* $|C|$ of a configuration $C : V \to \mathbb{N} = \{0, 1, \ldots\}$ is its total number of pebbles $\sum_{v \in V} C(v)$. The *pebbling number* $\pi(G) = \max_{r \in V} \pi(G, r)$, where $\pi(G, r)$ is defined to be the minimum number s so that every configuration of size at least s is r-solvable. The problem of deciding whether a given configuration on a graph is r-solvable is NP-complete, even for diameter two graphs and planar graphs. The problem of deciding whether $\pi(G) \leq k$ is Π_2^{P}-complete.

The pebbling numbers for some classes of graphs can be computed in polynomial time. For example, those for diameter 2 graphs can be found in quartic time. Recently, we proved that pebbling numbers for split graphs can be computed in $O(n^{1.41})$ time. We also conjectured that the pebbling number of a chordal graph of bounded diameter can be computed in polynomial time.

Along these lines, in this paper we study 2-paths, the sub-class of 2-trees whose graphs have exactly two simplicial vertices, as well as what we call semi-2-trees, the sub-class of 2-trees, each of whose blocks are 2-paths, and prove an exact formula that can be computed in linear time.

Determining optimum tree t-spanners for split graphs and cographs

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Keywords: tree t-spanners, threshold graphs, split graphs, cographs

Let G be a graph. A tree t-spanner of G is a spanning tree T in which the maximum distance in T between any pair of adjacent vertices of G is, at most, t. The MINIMUM STRETCH SPANNING TREE PROBLEM (MSST) for a graph G is the min-max problem of determining the minimum value $\sigma_T(G) = t$ such that G has a tree t-spanner. The MSST has been usually studied by establishing lower and upper bounds [4], and presenting the exact minimum value of $\sigma_T(G)$ for some graph classes [3]. There are also studies on computational complexity, for instance, determining whether $\sigma_T(G) = 2$ was settled to be a polynomially time solvable problem [1], while deciding if $\sigma_T(G) \geq 4$ is an NP-complete problem. Even so, determining whether $\sigma_T(G) = 3$ still remains open [2].

We present efficient algorithms to obtain $\sigma_T(G)$ values when G is a threshold graph, a split graph and a generalized octahedral graph. With this last result in addition to the tree decomposition of a cograph, we are able to prove exact $\sigma_T(G)$ values for cographs, a well known subclass of perfect graphs.

References

[1] Bondy, J. A. Trigraphs. Annals of Discrete Mathematics, v. 43 (1989): 69-79.

[2] Leizhen, C., and Corneil, D. G. Tree spanners. SIAM Journal on Discrete Mathematics, v. 8.3 (1995): 359-387.

[3] Fomin, F. V., Golovach, P. A., and Van Leeuwen, E. J. Spanners of bounded degree graphs. Information Processing Letters, v. 111.3 (2011): 142-144.

[4] Hassin, R., and Tamir, A. On the minimum diameter spanning tree problem. Information Processing Letters, v. 53.2 (1995): 109-111.

A kernelization algorithm for Closest String parameterized by the number of input strings

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Keywords: closest string; FPT; fixed-parameter; kernel; dynamic programming

Closest String is a well-known NP-hard problem and consists of finding a string that minimizes the maximum Hamming distance from a given set of strings. In 2003, Gramm, Niedermeier, and Rossmanith [1] presented an ILP formulation for Closest String using O(B(k) * k) variables, where k is the number of input strings and B(k) is the k-th Bell number. Such a formulation combined with Lenstra's [3] result for Integer Programming parameterized by the number of variables provides an FPT-algorithm for Closest String.

Although discrete parameterized algorithms for Closest String have been recently developed for different parameters [1, 2], to the best of our knowledge only FPT-algorithms based on integer linear programming is known for Closest String parameterized by k (cf. [1]), and no kernelization algorithm have been provided for this parameterized problem.

The goal of this paper is to present new combinatorial FPT-algorithms to solve Closest String. We present a kernelization algorithm which can be performed in linear time, and a dynamic programming that combined with our kernelization provides a great improvement on the running time required to solve Closest String parameterized by the number of input strings.

References

[1] J. Gramm, R. Niedermeier, P. Rossmanith, et al: Fixed-parameter algorithms for closest string and related problems. *Algorithmica* **37** 25–42, 2003.

[2] Z.-Z. Chen, B. Ma, and L. Wang.: A three-string approach to the closest string problem. *Journal of Computer and System Sciences* **78** 164–178, 2012.

[3] J. H. W. Lenstra.: Integer programming with a fixed number of variables. *Mathematics of Operations Research* **8** 538-548, 1983.

Prismas complementares com 2-atribuição de papéis

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Keywords: role assignment, atribuição de papéis, prisma complementar.

Atualmente, na era da internet e das redes sociais, os grafos são muito importantes para descrever a topologia das redes e representar as relações entre pessoas. Na teoria das redes sociais, uma sociedade é muitas vezes representada por um grafo simples, no qual os vértices representam indivíduos e as arestas representam as relações entre os indivíduos. A descrição da rede social é simplificada pela atribuição de papéis para os indivíduos, de modo que a relação de vizinhança entre os vértices seja preservada. Neste contexto, temos uma atribuição de papéis de um grafo simples, chamado convidado, para um grafo sem arestas múltiplas, chamado anfitrião, se existe um homomorfismo localmente sobrejetor, ou seja, um mapeamento de vértices do grafo convidado para o grafo anfitrião de modo que a relação de vizinhança é mantida. Assim, todos os papéis da vizinhança da imagem de um vértice aparecem como papéis da vizinhança do mesmo no grafo convidado.

Restringimos este trabalho ao caso do grafo anfitrião ter apenas dois vértices. Mesmo neste caso o problema da existência de uma atribuição de papéis, chamada de 2-atribuição, foi demonstrado ser NP-completo por Roberts e Sheng em 2001. Consideramos a classe dos prismas complementares, que são os grafos formados a partir da união disjunta do grafo com seu respectivo complemento, adicionando as arestas para um emparelhamento perfeito entre seus vértices correspondentes. Neste trabalho, temos como resultado a caracterização da 2-atribuição de papéis em prismas complementares feita para cada grafo anfitrião específico. Concluímos que qualquer prisma complementar de um grafo, que não seja o prisma do caminho com três vértices, tem uma 2-atribuição de papéis.

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A forbidden subgraph characterization of nested and 2-nested graphs†

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Keywords: forbidden induced subgraph, nested, 2-nested, split, C1P

We say a $(0,1)$-matrix is *nested* if it has the consecutive ones property for the rows (C1P) and every two rows are disjoint or nested (i.e., one is included in the other). We say a $(0,1)$-matrix is 2-*nested* if it has the C1P and admits a partition of its rows into two sets such that the submatrix induced by each of these sets is nested. We say a split graph G with split partition (K,S) is *nested* (resp. 2-*nested*) if the matrix $A(S,K)$ –which indicates the adjacency between vertices in S and K– is nested (resp. 2-nested). In this work, we characterize nested and 2-nested matrices by minimal forbidden submatrices. This implies a minimal forbidden induced subgraph characterization for nested and 2-nested graphs. Our result relies on a characterization of the consecutive ones property by minimal forbidden submatrices (Tucker, 1972).

Circle graphs (Even and Itai, 1971) are intersection graphs of chords in a circle. These graphs were characterized by Bouchet in 1994 by forbidden induced subgraphs of locally equivalent graphs. However, no complete characterizations of circle graphs by forbidden induced subgraphs of the graph itself is known. Nested and 2-nested graphs are common subclasses of the classes of threshold graphs and circle graphs. 2-nested graph characterization arises as a natural subproblem in our ongoing efforts to characterize those split graphs that are circle graphs by minimal forbidden induced subgraphs.

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On Gallai and anti-Gallai chordal graphs

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Keywords: Gallai graph, anti-Gallai graph, line graph, triangular line graph, chordal

For a graph G, the *Gallai graph* $\mathrm{Gal}(G)$ of G has the edges of G as its vertices, that is, $V(\mathrm{Gal}(G)) = E(G)$, and two distinct vertices e and f of $\mathrm{Gal}(G)$ are adjacent in $\mathrm{Gal}(G)$ if the edges e and f of G are adjacent but do not span a triangle in G. Gallai graphs were introduced in connection with cocomparability graphs [3] and were used in a polynomial-time recognition algorithm for claw-free perfect graphs [2]. Obviously, the Gallai graph $\mathrm{Gal}(G)$ is a spanning subgraph of the well-known line graph $L(G)$ of G, which is the intersection graph of the set of edges in G. The *anti-Gallai graph* $\Delta(G)$, or *triangular line graph* of G, is the complement of $\mathrm{Gal}(G)$ in $L(G)$, that is, it has $E(G)$ as vertex set and $E(L(G)) \setminus E(\mathrm{Gal}(G))$ as edge set. Gallai and anti-Gallai graphs were studied in [1, 5]. In [4], they characterize those graphs whose Gallai graphs are forests or trees.

In the present work, we prove that $\{G : \Delta(G) \in \{\text{chordal}\}\} = \{\text{partial 2-tree forests}\} + \{\text{isolated vertices}\}$ and characterize $\{G : \mathrm{Gal}(G) \in \{\text{chordal}\}\}$ by forbidden induced subgraphs.

[1] S. Aparna Lakshmanan, S. B. Rao, and A. Vijayakumar. Gallai and anti-gallai graphs of a graph. *Mathematica Bohemica*, 132(1):43–54, 2007.

[2] V. Chvátal and N. Sbihi. Recognizing claw-free perfect graphs. *Journal of Combinatorial Theory*, Series B 44(2):154 – 176, 1988.

[3] T. Gallai. Transitiv orientierbare graphen. *Acta Mathematica Academiae Scientiarum Hungarica*, 18(1):25–66, 1967.

[4] F. Joos, V. B. Le, and D. Rautenbach. Forests and trees among gallai graphs. *Discrete Mathematics*, 338(2):190–195, 2015.

[5] V. B. Le. Gallai graphs and anti-gallai graphs. *Discrete Mathematics*, 159(1):179 – 189, 1996.

Characterizing Star Graphs[1]

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Keywords: Star Graphs, Intersection Graphs, Edge Clique Covers

Biclique graphs were first characterized by Groshaus and Szwarcfiter [1]. A ***star*** is a complete bipartite graph $K_{1,r}$, for $r \geq 1$. The ***star graph*** of a graph H is the intersection graph of the family of maximal induced stars of H. An edge clique cover $\mathcal{Q} = \{Q_1, \ldots, Q_k\}$ of a graph G is ***star-partitioned*** if and only if each Q_i is partitioned as $Q_i \sim \{Q_i^c, Q_i^f\}$ and, $\forall a \in V(G)$, $\exists! i$ such that $a \in Q_i^c$ – in which case we say that $c(a) = i$. The ***cover*** of $a \in V(G)$ is defined as $Q(a) = \{i \mid a \in Q_i\}$ and $F(a) = Q(a) \setminus \{c(a)\}$. For each pair $Q_i, Q_j \in \mathcal{Q}$, define $\mathsf{ff}(i,j) = Q_i^f \cap Q_j^f$ and $\mathsf{cf}(i,j) = \left(Q_i^c \cap Q_j^f\right) \cup \left(Q_i^f \cap Q_j^c\right)$. All of the following edge clique covers of a graph G are star-partitioned.

Definition 1. An edge clique cover $\mathcal{Q}$ is ***compatible*** if, $\forall a \in V(G)$, $|Q(a)| \geq 2$ and if, $\forall \{Q_i, Q_j\} \subseteq \mathcal{Q}$, if $Q_i \cap Q_j \neq \emptyset$, either $\mathsf{cf}(i,j) = \emptyset$ or $\mathsf{ff}(i,j) = \emptyset$.

Definition 2. An edge clique cover $\mathcal{Q}$ is ***differentiable*** if $\forall Q_i \in \mathcal{Q}$ and $\forall \{a, a'\} \subseteq Q_i$ the following conditions hold:

1. If $\{a, a'\} \subseteq Q_i^c$, $\exists Q_j, Q_k \in \mathcal{Q}$ such that $a \in Q_j^f$, $a' \in Q_k^f$, $a \notin Q_k^f$, $a \notin Q_j^f$ and $\mathsf{cf}(j,k) \neq \emptyset$. Moreover, if $Q_i^c \cap Q_j^f \cap Q_k^f = \emptyset$, $\mathsf{cf}(j,k) \neq \emptyset$.
2. If $a \in Q_i^c$, $a' \in Q_k^c$ and $a \notin Q_k^f$, $\exists j \in F(a)$ with $\mathsf{cf}(j,k) \neq \emptyset$, $j \notin Q(a')$ and, $\forall j' \in F(a)$ with $\mathsf{cf}(j',k) = \emptyset$, $Q_i^c \cap \bigcap_{j'} \mathsf{ff}(j',k) \neq \emptyset$.
3. If $a \in Q_i^c$, $a' \in Q_k^c$ and $a \in Q_k^f$, for every $j \in F(a) \setminus \{k\}$, $\mathsf{cf}(j,k) = \emptyset$.
4. If $\{a, a'\} \subseteq Q_i^f$ and $j = c(a) \neq c(a') = k$, then either $Q_i^c \cap \mathsf{ff}\,(j,k) \neq \emptyset$ or $\mathsf{cf}\,(j,k) \neq \emptyset$.

Theorem 1. *G is the star graph for some graph H if and only if there is a compatible and differentiable star-partitioned edge clique cover $\mathcal{Q}$ of G.*

References

[1] Groshaus, M., and Szwarcfiter, J.L., *Biclique graphs and biclique matrices*, Journal of Graph Theory, **63(1)**, (2010), 1–16.

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New results on dually-CPT graphs

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Keywords: containment model, posets, dually-CPT graphs

A set family $\mathcal{F}$=$(F_x)_{x \in X}$ is a *containment model* of a poset $\mathbf{P}$= (X, P) if each element x can be assign to a set F_x in such a way that $x < y$ in $\mathbf{P}$ if and only if F_x is a proper subset of F_y. For instance, if a poset admits a containment model, where each set of the family is an interval of the line, then we will say it is a *containment order of intervals*, we will write CI poset for short.

A poset $\mathbf{P}$ is a *containment order of paths in a tree*, or CPT poset for short, if admits a containment model where every M_x is a path of a tree T. The *comparability graph* of $\mathbf{P}$ is the simple graph $\mathbf{G_P} = (X, E)$ where $xy \in E$ if and only if $x < y$ in $\mathbf{P}$ or $x > y$ in $\mathbf{P}$. Two posets are *associated* if their comparability graphs are isomorphic.

If $\mathbf{P}$ and its dual $\mathbf{P^d}$ are CPT we say that $\mathbf{P}$ is *dually-CPT*. If $\mathbf{P}$ and every other poset associated with $\mathbf{P}$ is CPT we say that $\mathbf{P}$ is *strong-CPT*. Clearly every strong-CPT poset is dually-CPT. The *strong-CPT* and *dually-CPT* graphs are the comparability graphs of the strong-CPT and dually-CPT posets, respectively.

In this work we prove, using the modular decomposition, that dually-CPT and strong-CPT graph classes are equal. As a corollary we obtain that the property of being strongly CPT is hereditary.

Vector Domination Problem on the family of Split-Indifference Graphs

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Keywords: Vector Domination Problem, Split-Indifference Graph, Algorithm

Given a graph $G = (V, E)$, a vector of integer requirements $R = \{R[v] \in \{0, 1, \ldots, deg(v)\} : v \in V\}$, and a natural number k, the Vector Domination Problem aims at determining if there is a set $S \subset V$ of size k such that each vertex $v \in V$ either is in S or has at least $R[v]$ neighbors in S.

The class of Split-Indifference graphs is formed by the intersection of the class of Split Graphs and the class of Proper Interval Graphs, that is, satisfies at the same time the intrinsic restrictions of both graph classes. An interesting formal definition of the class of Split-Indifference graphs found in the literature was proposed by [Ortiz et al. 1998] and proves the existence of four different cases of Split-Indifference graphs.

The proposed algorithm for those graphs is based on the theorem proposed by [Ortiz et al. 1998] since for each possible case a different method is applied. Let $G = (V, E)$ be a split-indifference graph and R the requirement vector, and let S be the R-dominating set that is being constructed for G.

The first case of Split-Indifference graph can be trivially solved: initially, the vertices are ordered by their requirements in ascending order. Next, the algorithm runs through this order and at each iteration a vertex $x \in V$ is analyzed. If $R[x] > |S|$ then x is included in S. Otherwise, x is dominated by S and the algorithm stops (all remaining vertices are dominated by S).

For the remaining cases of split-indifference graphs, the algorithm performs an isolated study for each possibility about the presence of the simplicial vertices in the R-dominating set. That is, for example, in the third case of Split-Indifference graph, there are two simplicial vertices v and w. In that case, the algorithm analyses four options: a) $v \notin S$ and $w \notin S$; (b) $v \in S$ and $w \notin S$; (c) $v \notin S$ and $w \in S$; (d) $v \in S$ and $w \in S$. In each case, the remaining clique (without simplicial vertices) is solved using the method described for the first case.

There is just one case that demands a different approach. On the fourth case of split-indifference graph, the algorithm cannot analyze the case where v and w are not included in S because when neighbors of v are included in S the requirement of w is also updated. In other to solve this case, a new approach was developed.

References

[Ortiz et al. 1998] Ortiz, C. Z., Maculan, N., and Szwarcfiter, J. L. (1998). Characterizing and edge-colouring split-indifference graphs. *Discrete Applied Mathematics*, 82(1-3):209–217.

Determinant families of dually chordal graphs

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Keywords: dually chordal graph, determinant family, clique, neighborhood, spanning tree, Helly property

A graph is *dually chordal* if it is the clique graph of some chordal graph. One characterization of dually chordal graphs is by means of the *compatible tree*. A spanning tree T of a graph G is a compatible tree if every maximal clique of G induces a subtree of T. A graph is dually chordal if and only if it has a compatible tree.

It is not difficult to see that compatible trees can also be characterized as those spanning trees for which every closed neighborhood of the graph induces a subtree.

The goal of this presentation is a generalization of the previous properties by the introduction of the concept of *determinant families*. Given a dually chordal graph G and a family $\mathcal{F}$ of subsets of $V(G)$, we say that $\mathcal{F}$ is *determinant* if the compatible trees of G are just those spanning trees for which every $F \in \mathcal{F}$ induces a subtree of T.

We find conditions to decide whether a family is determinant and we observe that some determinant families are stronger in the sense that the condition that T is a spanning tree in the definition can be dropped.

Finally, we apply this theory to generalize some characterizations of dually chordal graphs, like the ones that involve maximum weight spanning trees and the one that says that a graph G is dually chordal if and only if G is clique-Helly and $K(G)$ is chordal.

Characterization by forbidden subgraphs of near-bipartite P_4-tidy graphs

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Keywords: P_4-tidy, near-bipartition, partition problems

In this work, we will consider a graph partitioning problem which aroused much interest due to research in perfect graphs. It was introduced by Yang & Yuan [1] as *near-bipartite graphs* so that a graph can be partitioned on a independent set (a graph with no edges) and a forest (an acyclic and connected graph). There is also showed that this problem is NP-complete for general graph classes even with small degree (at least 4), small diameter (exactly 3)[2] and for perfect graphs.

Our main contribution is a characterization by forbbiden subgraphs, as stated on Theorem 1, for the class of near-bipartite graphs when restricted to the P_4-tidy graph class (for any induced $P_4 = H$ of the graph, there is at most one vertex out of H that induces a P_4 together with another three vertices in H). This class contains a cycle with 5 vertices (C_5) as an induced subgraph, thus, it is not contained in the class of perfect graphs.

Theorem 1 *Let G be a P_4-tidy graph. G is a near-bipartite graph if, and only if, it does not contains any of the graphs K_4 (complete graph with 4 vertices), $I_2 + I_2 + I_2$ (three pairs of trivial vertices totally adjacents) or W_5 (a C_5 and one vertex totally adjacent to the C_5) as an induced subgraph.*

References

[1] A. Yang and J. Yuan, "Partition the vertices of a graph into one independent set and one acyclic set," *Discrete mathematics*, vol. 306, no. 12, pp. 1207–1216, 2006.

[2] M. Bonamy, K. K. Dabrowski, C. Feghali, M. Johnson, and D. Paulusma, "Independent feedback vertex sets for graphs of bounded diameter," *Information Processing Letters*, vol. 131, pp. 26–32, 2018.

Characterizing General Fullerene Graphs[1]

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Keywords: $(3,4,6)$-*Fullerene Graphs, Combinatorial Curvature, Bipartite Edge Frustration.*

Fullerene graphs are 3-connected, planar and cubic graphs. They model fullerene molecules, which makes them mathematical objects widely studied and with a high degree of applicability. An inherent characteristic of fullerene graphs is that they are formed only by pentagonal faces (with combinatorial curvature equal to one) and hexagonal faces (flat).

From fullerene graphs, we proposed the construction of *general fullerene graphs*, containing four edges for each of its vertices and preserving faces of zero and one curvature. A general fullerene graph is a 3-connected, planar, 4-regular graph whose faces have size 3 or 4 (triangular and quadrangular faces, respectively). We are interested in investigating the relationship between these graphs and specific kind of molecules. The degree of stability of fullerene molecules can be measured by studying the fullerene graph stability. Many studies on graph stability refer to the parameter strictly related to the degree of bipartivity of a graph, that is, how different this graph is from its corresponding maximum bipartite spanning subgraph. The *bipartite edge problem* is to find the smallest number of edges that have to be deleted from a graph to obtain a bipartite spanning subgraph.

In this work, we determine an upper bound to the bipartite edge problem for general fullerene graphs by studying its dual problem, which corresponds to determining the minimum number of edges to be deleted from the dual graph so that all the degrees of the vertices of the remaining graph are even.

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On the adjacent vertex-distinguishing total coloring of power of cycles†

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Keywords: adjacent vertex-distinguishing total coloring, power of cycles

The adjacent vertex-distinguishing total coloring (AVDTC) of a graph G is a (proper) total coloring such that any two adjacent vertices have distinct sets of colors appearing on the vertex and its incident edges. The minimum number of colors required for an adjacent-vertex-distinguishing total coloring of a graph G is denoted by $\chi_{at}(G)$. In 2005, Zhang et al. proposed the AVDTC conjecture: for any simple graph G, $\chi_{at}(G) \leq \Delta(G) + 3$. This conjecture was verified for some special classes of graphs, however, the general case remains open. In this work, we study the AVDTC Conjecture for Power of Cycles. A related result, in the context of (proper) total coloring, was obtained by Campos and Mello [1]. Our results follow below.

Theorem 1. *Given an integer $n \geq 6$, if $G = C_n^2$, then $\chi_{at}(G) = 6$.*

Theorem 2. *Given two positive integers k and n such that $k < \lfloor n/2 \rfloor$, the AVDTC Conjecture holds for $G = C_n^k$, whenever n is even and $n \equiv r(mod\ k+1)$ for $r = 0, k-1, k$.*

Theorem 1 shows an explicit construction of an AVDTC with six colors. Papaioannou and Raftopoulou [2] proved the AVDTC Conjecture for 4-regular graphs, in particular, they show that $\chi_{at}(C_n^2) \leq 7$. So, we remark that Theorem 1 gives a tight bound in this particular case.

References

[1] Campos, C.N. and Mello, C.P. A result on the total colouring of powers of cycles. Discrete Applied Mathematics, 155 (2007) 585–597.

[2] Papaioannou, A. and Raftopoulou, C. On the AVDTC of 4-regular graphs, Discrete Mathematics 330 (2014) 20–40.

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Identifying codes in complementary prisms

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Keywords: Graph theory, Identifying codes, Complementary prisms

An *identifying code* of a graph is a dominating set that also has the property that the closed neighborhood of each vertex in the graph has a distinct intersection with the set. A graph admitting an identifying code is said to be *identifiable*. Identifying codes were introduced by Karpovsky, Chakrabarty and Levitin [2] and a range of applications may be found in the literature. The problem of finding minimum identifying codes was proven by Charon, Hudry and Lobstein to be NP-Complete [1], and therefore many authors have directed their study of identifying codes for restricted classes of graphs, such as paths, cycles, and some types of products of graphs.

In this work, we study identifying codes in complementary prism graphs. We give necessary and sufficient conditions for the complementary prism to be identifiable. We determine the minimum identifying code for specific graphs and characterize the complementary prisms with small identifying code numbers. We also present an upper bound for the complementary prism of connected graphs and prove it is sharp by showing that there are infinitely many graphs that equals the bound proposed.

References

[1] I. Charon, O. Hudry, and A. Lobstein, Minimizing the size of an identifying or locating-dominating code in a graph is NP-hard, Theor. Comput. Sci. 3 (2003) 2109-2120.

[2] M.G. Karpovsky, K. Chakrabarty, and L.B. Levitin, On a New Class of Codes for Identifying Vertices in Graphs, IEEE Transactions on Information Theory 44 (1998) 599-611.

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On identifying codes in the Cartesian product of a star and a path

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Keywords: Graph theory, Identifying codes, Cartesian product

An *identifying code* of a graph is a dominating set that also has the property that the closed neighborhood of each vertex in the graph has a distinct intersection with the set. Identifying codes were introduced by Karpovsky, Chakrabarty and Levitin [2] and a range of applications may be found in the literature. The problem of finding minimum identifying codes was proven by Charon, Hudry and Lobstein to be NP-Complete [1], and therefore many authors have directed their study of identifying codes for restricted classes of graphs, such as paths, cycles, and some types of products of graphs, including direct product, lexicographic product, corona product and Cartesian product.

In this work, we study identifying codes in the Cartesian product of a star and a path. We determine the size of minimum identifying codes for specific graphs within this class and we show upper and lower bounds on minimum identifying codes of such graphs, providing examples that attain the bounds proposed.

References

[1] I. Charon, O. Hudry, and A. Lobstein, Minimizing the size of an identifying or locating-dominating code in a graph is NP-hard, Theor. Comput. Sci. 3 (2003) 2109-2120.

[2] M.G. Karpovsky, K. Chakrabarty, and L.B. Levitin, On a New Class of Codes for Identifying Vertices in Graphs, IEEE Transactions on Information Theory 44 (1998) 599-611.

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Sobre Códigos Corretores de Distância Hamming 3

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Keywords: algoritmos, códigos Hamming, complexidade computacional

Um código linear é dito *ótimo* se, fixados uma distância Hamming d e um comprimento n, possui o maior número de palavras M possível. O problema de decodificação de um código linear geral é NP-completo [1]. Os códigos lineares ótimos para distância 3 são exatamente os códigos Hamming e os códigos Hamming encurtados. Embora esses resultados sejam antigos, não identificamos na literatura algoritmos eficientes para decodificação de tais códigos. O que se encontra, em geral, são códigos que procuram maximizar o valor de d para um dado n. Porém, seus algoritmos de decodificação não são adequados para valores grandes de n. Por exemplo, o melhor algoritmo de decodificação para os códigos Reed-Solomon tem complexidade $O(n^2)$, para os códigos Goppa, $O(n^3)$. O algoritmo básico de decodificação de um código linear, a decodificação por síndrome, tem complexidade $O(n\ 2^{n-k})$, onde k é a dimensão do código. Assim, em certas circunstâncias, pode ser mais vantajoso abrir mão de uma maior detecção de erros em nome de uma decodificação mais rápida, principalmente em sistemas de transmissão digital em que a relação sinal-ruído é alta. Desenvolvemos duas famílias de códigos Hamming encurtados, denotadas por $Gham(n)$ e $BP(n)$ e apresentamos seus processos de codificação e decodificação de complexidade $O(n)$.

Referências

[1] J. Bruck and M. Naor. The hardness of decoding linear codes with preprocessing. *IEEE Transactions on Information Theory*, 36(2):381–385, 1990.

The Biclique Graph of K_3-free Graphs are the Square of Some Graph

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Keywords: Bicliques, Biclique graph, Graph square

A *biclique* of a graph is a vertex set that induces a maximal complete bipartite subgraph. The *biclique graph* of a graph G, denoted by $KB(G)$, is the intersection graph of the bicliques of G. The biclique graph was introduced by Groshaus and Szwarcfiter [1], based on the concept of clique graphs. They gave a characterization of biclique graphs but it did not lead to a polynomial time recognition algorithm. Since then, the time complexity of the problem of recognizing biclique graphs remains open.

In this work we prove that every biclique graph of a K_3-free graph is the square of some graph. This result gives a tool for studying other classes of biclique graphs.

Let $P = X_P \cup Y_P$ and $Q = X_Q \cup Y_Q$ be two bicliques of a graph G, where $X_P \cap Y_Q = \emptyset$ and $X_Q \cap Y_P = \emptyset$. We say that P and Q are *mutually included* if $X_P \subset X_Q$ and $Y_Q \subset Y_P$. Given a graph G, define the graph $KB_m(G)$ as the graph with the bicliques of G as its vertex set and $\{P, Q\}$ is an edge if and only if P and Q are mutually included. Note that $KB_m(G) \subseteq KB(G)$.

We prove that for a K_3-free graph G, $KB(G) = (KB_m(G))^2$. And then, $KB(K_3\text{-free}) \subseteq (\mathcal{G})^2$, where $\mathcal{G}$ is the class of all graphs.

References

[1] M. Groshaus and J. L. Szwarcfiter. Biclique graphs and biclique matrices. *Journal of Graph Theory*, 63(1):1–16, 2010.

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On the Diameter of Spherical Fullerene Graphs†

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Keywords: diameter · Fullerene graphs

Fullerene graphs are mathematical models for molecules composed exclusively of carbon atoms, discovered experimentally in the early 1980s. Many parameters associated to these graphs have been discussed, trying to describe the stability of the fullerenes molecule. Formally, fullerene graphs are 3-connected, cubic, planar graphs with pentagonal and hexagonal faces.

Andova and Skrekovski (2012) conjectured a lower bound for the diameter of fullerene graphs. The relevance of this conjecture consists in the fact that it was conceived from perfectly spherical fullerene graphs which gives these graphs symmetry and, theoretically, high stability. We know that the curvature of fullerene graphs is given by their pentagonal faces, in this way fullerene icosahedral graphs preserve the same distance between their pentagonal faces. This distance between its pentagonal faces is characterized by two non-negative parameters i and j. An icosahedral fullerene graph $G - i, j$ is a graph in which its pentagonal (nearest) faces are $i + j$-units.

It is known that the Andova-Skrekovski conjecture is valid for the cases when $0 = i < j$, $0 < i = j$ and $j > \frac{11i}{2}$. In order to contribute with the study of this problem, in this work we verify the conjecture for the cases $j = i + 1$. Moreover, we present a lower bound for the diameter.

References

[1] Andova, V. and Škrekovski, R. Diameter of Fullerene Graphs with Full Icosahedral Symmetry, MATCH Communications in Mathematical and Computer Chemistry 70 (2012) 205–220.

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Some forbidden structures for the near-bipartition problem on distance-hereditary graphs

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Keywords: distance-hereditary, near-bipartition, partition problems

Interesting aspects of the near-bipartition problem of the class of distance-hereditary graphs are tackled in this study. Such partition problem was formulated by Yang & Yuan [3] as follows. The near-bipartition of a graph G consists in the finding of a partition $(\mathcal{S}, \mathcal{F})$ for the vertex set $V(G)$, in which $\mathcal{S}$ is a stable set and $\mathcal{F}$ induces an acyclic subgraph. Further, the authors in [1] demonstrated the problem is NP-complete for graphs with diameter equals to 3, degrees of 4 at most, and perfect graphs. In a corresponding way, we have verified some structures of distance-hereditary graphs do not admit partitions $(\mathcal{S}, \mathcal{F})$ for some candidates of vertices belonging to $\mathcal{S}$. Therefore, we are able to describe unlimited number forbidden distance-hereditary subgraphs not having the partitions $(\mathcal{S}, \mathcal{F})$. Our final contribution refers to establishing a sufficient condition, which uses the decomposition tree characterization as in [2], for ensuring the $(\mathcal{S}, \mathcal{F})$ partitioning of the subclass of distance-hereditary graphs.

References

[1] M. Bonamy, K. K. Dabrowski, C. Feghali, M. Johnson, and D. Paulusma. Independent feedback vertex sets for graphs of bounded diameter. *Information Processing Letters*, 131:26–32, 2018.

[2] M. Chang, S. Hsieh, and G. Chen. Dynamic programming on distance-hereditary graphs. In *Algorithms and Computation, 8th International Symposium, ISAAC '97, Singapore, December 17-19, 1997, Proceedings*, pages 344–353, 1997.

[3] A. Yang and J. Yuan. Partition the vertices of a graph into one independent set and one acyclic set. *Discrete mathematics*, 306(12):1207–1216, 2006.

Caracterização estrutural de grafos-$(1,2)$ bem-cobertos

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Palavras-chave: caracterização, grafos bem cobertos, partições

Um grafo é bem coberto quando todos os seus conjuntos independentes maximais possuem a mesma cardinalidade. O conceito de *grafos bem cobertos* foi introduzido em 1970 por M. Plummer [4]. Sabe-se que obter o número de independência de um grafo arbitrário é um problema NP-completo, entretanto, quando restrito a grafos bem cobertos, o problema é polinomial. Chvátal e Slater [3] mostraram que o reconhecimento de grafos bem cobertos é coNP-completo. Um grafo é (k,ℓ) se seu conjunto de vértices admite uma partição em k conjuntos independentes e ℓ cliques. Brandstädt [2] provou que o reconhecimento de grafos (k,ℓ) é polinomial quando $k \leq 2$ e $\ell \leq 2$, e NP-completo caso contrário. Alves et al. [1] provaram que o reconhecimento de um grafo-(k,ℓ) bem coberto para as partições $(1,0)$, $(0,1)$, $(2,0)$, $(0,2)$, $(1,1)$ e $(1,2)$ é polinomial, e que é difícil para todas as outras. Neste trabalho, caracterizamos os grafos-$(1,2)$ bem cobertos, isto é, grafos bem cobertos que admitem uma partição em um conjunto independente e duas cliques.

Referências

[1] Alves, S. R., Dabrowski, K. K., Faria, L., Klein, S., Sau, I., Souza, U. dos S., On the (Parameterized) Complexity of Recognizing Well-Covered (k,ℓ)-graphs, COCOA 2016, 8, 2016, LNCS (2016), 423–437.

[2] Brandstädt, A., Partitions of graphs into one or two independent sets and cliques, Discrete Mathematics **152** (1996), 47–54.

[3] Chvátal, V. and Slater, P. J., A Note on Well-Covered Graphs, Annals of Discrete Mathematics, **55** (1993), 179–181.

[4] Plummer., M. D. Some covering concepts in graphs, Journal of Combinatorial Theory, vol. 8, pp. 91-98, 1970.

An optimal algorithm to totally color some power of cycle graphs

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Keywords: total coloring; total coloring conjecture; power of cycles

A total coloring of a graph G is a function which assigns colors to the vertices and edges of G, so that there are no conflict between adjacent elements. The *total chromatic number* of a graph G, denoted by $\chi_T(G)$, is the minimum number of colors needed to totally color G. A well-known bound is $\chi_T(G) \geq \Delta(G) + 1$, where $\Delta(G)$ represents the maximum degree of a vertex in G. The *total coloring conjecture* (TCC) was proposed independently by Behzad (1967) and Vizing (1964) and states that, for every simple graph G, $\chi_T(G) \leq \Delta(G) + 2$. This conjecture remains open to regular, chordal and power of cycle graphs. If $\chi_T(G) = \Delta(G) + 1$, then G is said to be *type 1*. If $\chi_T(G) = \Delta(G) + 2$, then G is said to be *type 2*. Deciding if a graph is type 1 is a NP-complete problem, and remains NP-complete even if G is a bipartite regular graph.

The power of cycle graph C_n^k has C_n as spanning subgraph and additional edges between vertices at distance at most k in C_n.

The TCC was proved to C_n^k, when n is even [1]. For fixed value of k, the TCC was proved to C_n^3 and C_n^4, and for C_n^2, the total chromatic number is established: C_7^2 is type 2, while C_n^2 is type 1 otherwise [2].

In the present work, we prove that, except for the complete graphs K_4 and K_6, all power of cycle graphs C_n^3 with even n are type 1. Our proof is constructive, in the sense that we develop an algorithm that optimally totally color power of graphs C_n^3 with even n.

References

[1] C.N. Campos and C.P. de Mello: A result on the total colouring of powers of cycles. *Discrete Applied Mathematics* **155** (2007) 585–579.

[2] C.N. Campos: O problema da coloração total em classes de grafos. *PhD thesis*, Universidade Estadual de Campinas - Unicamp (2006).

Equitable total coloring of graphs with universal vertex

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Keywords: equitable total coloring, equitable total chromatic number, graphs with universal vertex

A total coloring of a graph G is an assignment of colors to the elements of G, vertices and edges, such that every pair of adjacent or incident elements have distinct colors. The minimum number of colors for a total coloring of G is the total chromatic number of G, $\chi''(G)$. Similarly, an edge (vertex) coloring of a graph G is an assignment of colors to the edges (vertices) of G such that any two adjacent edges (vertices) have distinct colors.

A color class c is composed by the elements of G with color c. An equitable coloring of G is a coloring such that the cardinality of any two color classes differs by at most one. The least number of colors for an equitable total coloring of a graph G is the equitable total chromatic number of G, $\chi''_e(G)$. De Werra shows that if there is an edge coloring of graph G using k colors, then G also has an equitable edge coloring using k colors. Hajnal and Szemerédi stated that a graph G has an equitable vertex coloring using k colors for each $k \geq \Delta(G)+1$. Hung-Lin Fu conjectured that for every graph G, there is an equitable total coloring using k colors for each $k \geq \max\{\chi''(G), \Delta(G)+2\}$.

A universal vertex v is a vertex of G with degree $|V(G)|-1$. Considering a graph G with at least one universal vertex, we show that $\chi''_e(G) = \Delta(G)+1$ when $|V(G)|$ is odd or when $|V(G)|$ is even and $|E(\overline{G})| \geq \frac{|V(G)|}{2}$, otherwise we show an equitable total coloring of G using $\Delta(G)+2$ colors. Therefore, the Conjecture of Fu holds for graphs with universal vertex.

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A Recolouring Procedure for Total Colouring

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The Total Colouring Conjecture states that every (simple) graph $G = (V, E)$ with maximum degree Δ admits a $(\Delta + 2)$-total colouring. This conjecture was proposed independently by M. Behzad and V. G. Vizing 50 years ago and, since then, some upper bounds for $\chi''(G)$ (the total chromatic number of G) have been found, being $\Delta + 10^{26}$ the only one of the form $\Delta + C$ for a fixed constant C, a result by M. Molloy and B. Reed of 1998.

This work presents a polynomial-time heuristic for constructing edge by edge a $(\Delta + 2)$-total colouring of G over an initial $(\Delta + 2)$-vertex-colouring (recall that $(\Delta + 1)$-vertex-colourings can be greedily constructed). Our algorithm is based on a recolouring procedure similar to Vizing's recolouring procedure for edge-colouring. We show that if there is some $uv \in E$ such that $G - uv$ has a $(\Delta + 2)$-total colouring φ and there is a *complete recolouring fan* for uv, as we define in the sequel, then G is also $(\Delta + 2)$-total colourable.

A *recolouring fan* for uv is a sequence $v_0, \ldots, v_k$ of distinct neighbours of u such that $v_0 = v$ and, for all $i \in \{0, \ldots, k-1\}$, the colour $\alpha_i := \varphi(uv_{i+1})$ is *missing* at v_i, i.e. it colours no element of $S(v_i) := \partial_G(v_i) \cup \{v_i\}$. Let $\mathscr{C}$ be the set of the $\Delta + 2$ colours used. The fan is *complete* if either (i) there is some $\beta \in \mathscr{C}$ missing at both u and v_k, or (ii) for some $\alpha \in \mathscr{C}$ missing at v_k and some $\beta \in \mathscr{C}$ missing at u, the α-coloured element of $S(u)$ and the β-coloured element of $S(v_k)$ are not in the same component C of the subgraph of the total graph of G induced by the elements coloured with α or β (the total graph of G is the graph whose vertex set is $V(G) \cup E(G)$ and whose edges represent the adjacencies and incidences between the elements of G). If (ii) holds, exchanging the colours of the elements of C brings us to (i).

Since our recolouring procedure uses $\Delta + 2$ colours, we aim to investigate graph classes in which it may lead to novel results on total colouring. It would also be interesting to verify how our procedure behaves empirically.

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The b-continuity of graphs with large girth

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Let G be a simple graph. A *k-coloring* of G is a function $\psi\colon V(G) \to \mathbb{N}$ such that $|\psi(V(G))| = k$ and $\psi(u) \neq \psi(v)$ whenever $uv \in E(G)$. We say that $u \in V(G)$ is a *b-vertex in ψ (of color $\psi(u)$)* if for every $c \in \psi(V(G)) \setminus \{\psi(u)\}$, there exists a vertex v adjacent to u such that $\psi(v) = c$. If $c \in \psi(V(G))$ is a color class that has no b-vertex on ψ, then we can separately change the color of each vertex w in $\psi^{-1}(c)$ for a color in $\psi(V(G)) \setminus \{c\}$ and obtain a proper coloring with fewer colors. If ψ is a coloring such that we cannot apply this algorithm to decrease the amount of used colors, then every color class contains a b-vertex. Such coloring is called a *b-coloring*. As the coloring problem is NP-complete, not always a b-coloring uses only $\chi(G)$ colors. The *b-spectrum of G*, denoted by $S_b(G)$, is the set of integers k such that G has a b-coloring with k colors. The *b-chromatic number of G*, denoted by $b(G)$, is the maximum element of $S_b(G)$. We say that G is *b-continuous* if its b-spectrum contains all integers from $\chi(G)$ to $b(G)$, i.e., $S_b(G) = [\chi(G), b(G)] \cap \mathbb{Z}$. An infinite number of graphs that are not b-continuous is known. It is also known that for each subset $S \subset \mathbb{N} - \{1\}$, there exists a graph G_S such that $S_b(G_S) = S$. Several results suggest a strong link between b-colorings and high girth. It has been proven that if G has girth at least 7, then G has b-chromatic number at least $m(G) - 1$. More recently it has been proven that graphs with girth at least 10 are b-continuous. In this article, we prove that if G has girth at least 8, then G is b-continuous and that if G has girth at least 7, then $[2\chi(G), b(G)] \cap \mathbb{Z} \subset S_b(G)$.

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On the null structure of bipartite graphs without cycles of length multiple of 4

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Keywords:Bipartite Graphs, Independence number, Matching number, Maximum matchings, Independent sets, Eigenvectors, Null space

We say that a bipartite graph without cycles of length multiple of 4 is a BC_{4k}-free graphs. In this work we study the null space of BC_{4k}-free graphs, and its relation to structural properties. We decompose them into two different types of graphs: N-graphs and S-graphs. N-graphs are graphs with a perfect matching (the order of the graph is twice its matching number). S-graphs are graphs with a unique maximum independent set. We relate the independence number and the matching number of a BC_{4k}-free graph with its N-graph and its S-graph. Among other results, we show that the rank of a BC_{4k}-free graph is twice its matching number, generalizing a result for trees due to Bevis et al [1]. About maximum independent sets, we show that the intersection of all maximum independent sets of a BC_{4k}-free bipartite graph coincides with the support of its null space.

References

[1] Bevis, Jean H and Domke, Gayla S and Miller, Valerie A. Ranks of trees and grid graphs. *J. of Combinatorial Math. and Combinatorial Computing*, 18: 109–119, 1995.

Proper gap-labellings of unicyclic graphs

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Let $G = (V, E)$ be a connected simple graph with $|V| \geq 2$. A *gap*-$[k]$-*edge-labelling* of G is a pair (π, c_π) where $\pi : E \to \{1, \ldots, k\}$ is an edge-labelling of G and $c_\pi : V \to \{0, \ldots, k\}$ is a proper vertex-colouring of G such that, for every $v \in V$, colour $c_\pi(v)$ is: (i) 1 if $d(v) = 1$; and (ii) $\max_{uv \in E}\{\pi(uv)\} - \min_{uv \in E}\{\pi(uv)\}$ if $d(v) \geq 2$. Colour c_π is *induced* by the largest *gap* among the labels of its incident edges. Analogously, a *gap*-$[k]$-*vertex-labelling* of G is a pair (π, c_π), $\pi : V \to \{1, \ldots, k\}$ and c_π, a proper vertex-colouring of G, with $c_\pi(v)$ induced by the largest gap among the labels of $N(v)$ when $d(v) \geq 2$, and 1, otherwise. The least k for which G *admits* a gap-$[k]$-edge-labelling (gap-$[k]$-vertex-labelling) is denoted by $\chi^{\mathrm{g}}_{E}(G)$ ($\chi^{\mathrm{g}}_{V}(G)$). Gap-$[k]$-edge-labellings were proposed, in 2012, by Tahraoui, Duchêne and Kheddouci. Since then, several works have both established general bounds for $\chi^{\mathrm{g}}_{E}(G)$ and determined it for classes of graphs. Brandt, Moran, Nepal, Pfender and Sigler (2016) proved that $\chi^{\mathrm{g}}_{E}(G) \in \{\chi(G), \chi(G) + 1\}$ unless G is a star, in which case $\chi^{\mathrm{g}}_{E}(G) = 1$. The vertex variant was introduced by Dehghan, Sadeghi and Ahadi in 2013. In their work, the authors study the algorithmic complexity of decision problems associated with these labellings, proving that: determining whether graphs admit gap-$[k]$-edge-labellings and gap-$[k]$-vertex-labellings is NP-complete when $k \geq 3$. For $k = 2$, both variants are NP-complete for bipartite graphs. However, there exist some classes of graphs for which these problems are polynomially solvable. Particularly for planar bipartite graphs with $\delta(G) \geq 2$, the edge variant is in P, whereas if degree-one vertices are admitted, the problem remains NP-complete. This implies that degree-one vertices may play an important role in the computational complexity of gap-labellings. In this context, Dehghan et al. determined $\chi^{\mathrm{g}}_{V}(G)$ for trees. Our work expands this result for unicyclic graphs, which are connected graphs with $|V| = |E|$. We studied both the edge and vertex variants of proper gap-labellings for unicyclic graphs. For this family, we determined that $\chi^{\mathrm{g}}_{V}(G) = \chi(G)$. Moreover, if the length of the cycle is odd, then $\chi^{\mathrm{g}}_{E}(G) = 3$.

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A Decomposition for Edge-colouring

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Keywords: edge-colouring, chromatic index, biconnectivity, graph algorithms

By the celebrated Vizing's Theorem of 1964, the chromatic index of a graph G with maximum degree Δ, denoted $\chi'(G)$, is either Δ or $\Delta + 1$ (in which case G is *Class 1* or *Class 2*, respectively) and a $(\Delta+1)$-edge-colouring of G can be computed in polynomial time. However, deciding if a given graph is *Class 1* is an NP-complete problem, as shown by Holyer in 1981.

It is clear that $\chi'(G)$ is the maximum amongst the chromatic indices of the connected components of the graph G. Further, we show that $\chi'(G)$ is the maximum amongst the degrees of the articulation points of G and the chromatic indices of its biconnected components. For this, it suffices to prove that if G_1 and G_2 are any two graphs with $V(G_1) \cap V(G_2) = \{u\}$, then $\chi'(G_1 \cup G_2) = \max\{\chi'(G_1), \chi'(G_2), d_{G_1 \cup G_2}(u)\} =: k$. The proof follows by taking k-edge-colourings for G_1 and G_2 using the same colour set $\mathscr{C}$ and then permuting $\mathscr{C}$ on G_2 so that the colours of the edges incident to u in $G_1 \cup G_2$ become all distinct.

Our result yields a decomposition heuristic for edge-colouring algorithms and implies that an optimal edge-colouring can be computed in polynomial time for graphs with m edges and $O(\log m)$-size biconnected components, using the algorithm by Björklund and Husfeldt of 2009 which gives an optimal edge-colouring of any graph with m edges in $O(2^m m^{O(1)})$ time. This motivates further investigation on phase diagrams for the size of the biconnected components in random graph models which capture the aspects of real-world networks. We also encourage future works to extend our proof for the case wherein $|V(G_1) \cap V(G_2)| \geq 2$, since this may lead to results on edge-colouring indifference graphs, for which only partial results are known, as those by Figueiredo, Meidanis, Mello, and Ortiz of 2003.

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The Colorability problem on (r,ℓ)-graphs and a few parametrized solutions

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Keywords: *colorability; graph coloring; (r,ℓ)-graph; list coloring; parametrized complexity*

An *(r,ℓ)-graph* is a graph that can be partitioned into r independent sets and ℓ cliques; In the k-Colorability problem we are asked to determine whether a given graph G admits a vertex coloring using at most k colors such that adjacent vertices have different colors.

In this work, we describe a *Poly vs NP-complete* dichotomy of this problem regarding to the parameter r and ℓ of (r,ℓ)-graphs, determining the boundaries of the NP-completenes for such a class. In addition, we analyze the complexity of the problem on (r,ℓ)-graphs under the parametrized complexity perspective.

A parameterized problem (Π, k) is said *fixed-parameter tractable* (FPT) if it can be solved in time $f(k) \times n^{O(1)}$, where f is an arbitrary function, and n is the size of the input.

Using a reduction from k-Colourability on (r,ℓ)-graph to List-Coloring as strategy, we are able to discovery that given a (2,1)-partition of the input graph G, to finding an optimal coloring of G is: W[1]-hard when parametrized by the size of the smallest independet part; Para-NP-complete when parametrized by the size of the complete part; FPT when parametrized by the number of vertices having no neighbors in the complete part; and FPT when the size of the complete part and the size of the smallest independent part are agregated parameters.

Colorings, Cliques and Relaxations of Planarity

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Keywords: chromatic number, crossing number, skewness, genus, thickness

A simple graph G is *planar* if it can be embedded in the plane, i.e., it can be drawn on the plane such that no edges cross each other. There are many relaxations of planarity, i.e. graph invariants that measure how close a graph is to a planar graph. Examples of relaxations of planarity include: the *crossing number* $cr(G)$ (the minimum number of edge crossings in any drawing of G in the plane), the *skewness* $\mu(G)$ (the minimum number of edges whose removal makes the graph planar), the genus $\gamma(G)$ (the minimum genus of the orientable surface on which G is embeddable), and the *thickness* $\theta(G)$ (the minimum number of planar subgraphs of G whose union is G.)

A conjecture by Albertson states that if $\chi(G) \geq n$ then $cr(G) \geq cr(K_n)$, where $\chi(G)$ is the chromatic number of G. This conjecture is still open for $n > 16$. In this paper we consider the statements corresponding to this conjecture where the crossing number of G is replaced with other relaxations of planarity such as $\mu(G)$, $\gamma(G)$, and $\theta(G)$.

First we show that for every simple graph G if $\chi(G) \geq n$ then $\mu(G) \geq \mu(K_n)$. The statement is equivalent to the Four Color Theorem when $n = 5$, and is equivalent to a generalization of the Five Color Theorem by Kainen when $n = 6$. We provide an elementary proof when $n \geq 7$.

Then we show that for every simple graph G if $\chi(G) \geq n$ then $\gamma(G) \geq \gamma(K_n)$.

Finally we consider the corresponding statement: if $\chi(G) \geq n$ then $\theta(G) \geq \theta(K_n)$. We show that this statement is true for infinitely many values of n, but not for all n. The Sulanke graph $K_{11} - C_5$ is a counterexample when $n = 9$. When $n = 10$, 11, or 12, determining the truth value of this statement is equivalent to Ringel's famous Earth-Moon problem.

Algoritmos para os Casos Polinomiais da Coloração Orientada

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PALAVRAS CHAVE: Coloração Orientada, Algoritmos Polinomiais.

Seja $\overrightarrow{G} = (V, A)$ um grafo orientado, $\overrightarrow{(x,y)}$, $\overrightarrow{(z,t)} \in A(\overrightarrow{G})$, e C um conjunto com k cores. Uma função ϕ de C para $V(\overrightarrow{G})$ tal que $\phi(x)$ é diferente de $\phi(y)$ e se $\phi(x) = \phi(t)$ então $\phi(y)$ é diferente de $\phi(z)$ é chamada de k-coloração orientada. O número cromático orientado $\chi_o(\overrightarrow{G})$ é o menor k tal que $\overrightarrow{G}$ admite uma k-coloração orientada.

O problema da k-coloração orientada pode ser visto como um homomorfismo de $\overrightarrow{G}$ em um grafo $\overrightarrow{T}$ com k vértices, podemos chamar o problema de colorir $\overrightarrow{G}$ com $\overrightarrow{T}$ de problema da $\overrightarrow{T}$-coloração. Bang-Jensen, et al. [1] demonstrou que o problema $\overrightarrow{T}$-coloração é polinomial quando $\overrightarrow{T}$ é um torneio acíclico ou que contém um único ciclo orientado.

Neste trabalho demonstramos que um grafo acíclico que não contém o caminho $\overrightarrow{P_{n+1}}$ como subgrafo pode ser colorido pelo torneio transitivo $\overrightarrow{T_n}$ com n vértices e que um grafo $\overrightarrow{G}$ que contém um único ciclo orientado de tamanho múltiplo de 3 pode ser colorido por um torneio que contém um único ciclo orientado. A partir destes sub-casos obtemos algoritmos para resolver todos casos em que o problema da $\overrightarrow{T}$-coloração é polinomial.

Apresentamos algoritmos polinomiais para os casos em que o $\chi_o(\overrightarrow{G}) \leq 3$. Demonstramos que um grafo $\overrightarrow{G}$ tem $\chi_o(\overrightarrow{G}) = 2$ se e somente $\forall x \in V(\overrightarrow{G})$, x é um vértice fonte ou x é um vértice sumidouro. Para grafos que tem $\chi_o(\overrightarrow{G}) \geq 4$ apresentamos uma relação do número de torneios em que o problema da $\overrightarrow{T}$-coloração é polinomial com o número de torneios em que o problema é NP-completo. Por fim apresentamos um algoritmo para a geração de torneios que contém apenas um ciclo orientado.

Referências

[1] Jørgen Bang-Jensen, Pavol Hell, and Gary MacGillivray. The complexity of colouring by semicomplete digraphs. SIAM *journal on discrete mathematics*, 1(3):281-298, 1988.

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Equitable total coloring of classes of tripartite complete graphs†

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Keywords: equitable total coloring · graph coloring · algorithm

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A *k-equitable total coloring* is an assignment of colors to the edges and vertices of a graph such that adjacent and incident elements receive different colors and the difference between the cardinalities of any two color classes is at most one. The smallest integer k for which a graph G has a k-equitable total coloring is called the *equitable total chromatic number* of G and it is denoted by $\chi''_e(G)$. A graph is said to be *complete tripartite* if its vertex set can be partitioned into 3 sets such that no two vertices within the same part are adjacent, and there is an edge between any two vertices of different parts of the partition. If each part of the partition has the same order then it is said to be *balanced*. Let Δ be the *maximum degree* of a graph. The Equitable Total Coloring Conjecture (ETCC) due to Wang [1] states that every simple graph G has $\Delta + 1 \leq \chi''_e(G) \leq \Delta + 2$. To contribute with the ETCC, we investigate equitable total colorings of complete tripartite non balanced graphs. Such graphs are denoted by $K_{a,b,c}$ meaning that the parts of the partition of the vertex set have, respectively, a, b and c vertices. We verify the ETCC for the following classes of complete tripartite (non balanced) graphs: $K_{a,b,c}$ with $a < b = c$; $a = b$ and $c \geq b^2$ if $b \neq 1$ or $c \geq 2$ if $b = 1$; and $a < b$ and $c \geq b^2$ has $\chi''_e = \Delta + 1$.

References

[1] W. F. Wang. *Equitable total coloring of graphs with maximum degree 3.* Graphs Combin. 18 (2002) 677–685.

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Graphs with small fall-spectrum

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Keywords: fall-coloring, fall-spectrum, perfect graphs, NP*-completeness.*

Given a proper coloring f of a graph G, a *b-vertex* in f is a vertex that is adjacent to every color class but its own, and f is a *fall-coloring* if every vertex is a b-vertex. The *fall-spectrum of* G is the set $\mathcal{F}(G)$ of all values k for which G admits a fall-coloring with k colors. Observe that a graph not always have such a coloring, i.e., $\mathcal{F}(G)$ can be empty; for instance, one can easily see that if $\chi(G) > \delta(G) + 1$, then $\mathcal{F}(G) = \emptyset$. These concepts were introduced by Dunbar et. al. in 2000, where they prove that deciding whether $\mathcal{F}(G) \neq \emptyset$ is NP-complete.
Some authors have found that some subclasses of perfect graphs have the property that: (*) $\mathcal{F}(G) \neq \emptyset$ if and only if $\chi(G) = \delta(G)+1$. This has led Kaul and Mitilos to conjecture that (*) holds for every perfect graph. We prove that this is not true by showing a chordal graph on which (*) does not hold. Note that, because $\delta(G) + 1$ is also an upper bound for the values in $\mathcal{F}(G)$, we get that (*) implies $\mathcal{F}(G) \subseteq \{\chi(G)\}$. This led us to the question about which kind of graphs have small fall-chromatic spectra, i.e., which graphs have the property $\mathcal{F}(G) \subseteq \{\chi(G)\}$. We prove that: 1. $\mathcal{F}(G) \subseteq \{\chi(G)\}$ for chordal graphs and P_4-sparse graphs; 2. deciding whether $\mathcal{F}(G) \neq \emptyset$ is NP-complete for chordal graphs; and 3. deciding whether $|\mathcal{F}(G)| > 1$ is NP-complete for bipartite graphs (deciding whether $\mathcal{F}(G) \neq \emptyset$ is trivial for bipartite graphs). The NP-completeness results discard the possibility of getting a characterization for these graphs. This is why we investigate perfectness aspects. A graph G is *all-defined fall-perfect* if $\mathcal{F}(H) = \{\chi(H)\}$ for every induced subgraph H of G; and it is *fall-perfect* if $\mathcal{F}(H) \subseteq \{\chi(G)\}$ for every induced subgraph H of G. We characterize the all-defined fall-perfect graphs and the bipartite fall-perfect graphs.

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Coloring Game: characterization of a $(3, 4^*)$-caterpillar†

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Keywords: Coloring Game; Game Chromatic Number; Caterpillar

The *coloring game* was conceived by Brams, firstly published in 1981 by Gardner, and reinvented in 1991 by Bodlaender, in the context of graphs. Alice and Bob take turns properly coloring the vertices of a graph, Alice trying to minimize the number of colors used, while Bob tries to maximize them. The *game chromatic number* is the smallest number of colors that ensures that the graph can be properly colored despite Bob's intention. We denote by $\chi_g^a(G)$ (or simply $\chi_g(G)$) the game chromatic number of G when Alice starts the game, and $\chi_g^b(G)$ when Bob does it. It is known that $\chi_g(F) \leq 4$, for F forest and it is an open problem to characterize which forests have $\chi_g(F) = 3$ or 4. In 2016, Furtado et al. contributed to this study by considering a special tree called *caterpillar*. They studied χ_g of three infinite classes of caterpillars: caterpillars with a maximum degree 3, without vertices of degree 2, and without vertices of degree 3. The only remaining case to conclude the study of game chromatic number of caterpillars is the case with vertices of degree 1, 2, 3 and at least 4. We contribute to this study by characterizing $(3, 4^*)$-caterpillars. We say that H is a $(3, 4^*)$-caterpillar when $\chi_g^a(H) = 3$ and H is minimal with respect to $\chi_g^b(H) = 4$. We conclude the characterization of $(3, 4^*)$-caterpillars with vertices of degree 1, 2, 3 and at least 4. This study is important because having two induced $(3, 4^*)$-subcaterpillars is a sufficient condition for any caterpillar H to have $\chi_g(H) = 4$.

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Matching problem for vertex colored graphs[1]

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Keywords: perfect matching, graph partition

A classical result in Graph Theory is Tutte's Perfect Matching Theorem which characterizes graphs admitting a perfect matching.

We present a generalization of this result to triples (V, E, c), where (V, E) is a graph and c is a function from V to the set of non negative integers. We call such triples *(vertex) colored graphs*. Tutte's Theorem will be the special case when every edge $uv \in E$ is *monochromatic*, that is, when $c(u) = c(v)$. Equivalently, whenever c is constant in each connected component of (V, E).

We define a *perfect matching* of a colored graph (V, E, c) to be a function m from E to the set of nonnegative integers such that, for each $v \in V$, the set $\{uv \in E : m(uv) = c(v)\}$ is a singleton.

We prove that a colored graph (V, E, c) has a perfect matching if and only if for each $S \subseteq V$ and each $F \subseteq E$ of non-monochromatic edges, the graph $(V, E) \setminus (S \cup F)$ has at most $|S \cup F|$ connected components with an odd number of vertices and such that each non-monochromatic edge incident to some vertex in the component belongs to F.

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Alguns Resultados em Coloração Orientada e Clique Coloração Orientada [1]

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Palavras Chave: Coloração orientada, clique coloração orientada.

Uma *k-coloração orientada* de um grafo orientado $\vec{G} = (V, \vec{E})$ é uma partição de V em k subconjuntos, tal que não existem dois vértices adjacentes pertencentes ao mesmo subconjunto, e todos os arcos entre dois subconjuntos tem a mesma orientação. Um *homomorfismo* de $\vec{G}_1$ em $\vec{G}_2$ corresponde a uma k-coloração orientada de $\vec{G}_1$ se $\vec{G}_2$ tem k vértices, $\vec{G}_2$ é chamado de *grafo de cor* para $\vec{G}_1$. O número cromático orientado $\chi_o(\vec{G})$ é o menor inteiro k, tal que $\vec{G}$ admita uma k-coloração orientada. Um resumo pode ser visto em Sopena [1]. Neste trabalho apresentamos o único torneio com 5 vértices que é subgrafo de qualquer grafo de cor para grafos planares e cúbicos, também provamos que $\chi_o(G \cup C) \leq 5$, onde $\chi_o(G) \leq 4$ e C é um ciclo.

Além disso, apresentamos a definição do número clique cromático orientado $\vec{\kappa}(G)$. Provamos que se todo hipergrafo orientado $\vec{\mathcal{H}}(G)$ é um grafo orientado, então $\vec{\kappa}(G) = \chi_o(G)$. Além disso, provamos que $\vec{\kappa}(K_n) = 2$ e $\vec{\kappa}(C_n) = \chi_o(C_n)$. Por fim, apresentamos a conjectura que $\vec{\kappa}(G) \leq \chi_o(G)$.

References

[1] É. Sopena, Homomorphisms and colourings of oriented graphs: An updated survey, *Disc. Math.* **339** (2016) 1993-2005.

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[1] Partially supported by CAPES, CNPq, FAPEG and FAPERJ.

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On the convexity number for complementary prisms [1]

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Keywords: geodetic convexity, convexity number, complementary prisms.

Let G be a finite, simple, and undirected graph and let S be a set of vertices of G. In the geodetic convexity, a set of vertices S of a graph G is *convex* if all vertices belonging to any shortest path between two vertices of S lie in S. The cardinality $con(G)$ of a maximum proper convex set S of G is the *convexity number* of G. The *complementary prism* $G\overline{G}$ of a graph G arises from the disjoint union of the graph G and $\overline{G}$ by adding the edges of a perfect matching between the corresponding vertices of G and $\overline{G}$. Dourado et al. [1] proved that the decision problem related to the convexity number is NP-complete even restricted to bipartite graphs, but it can be computed in linear time for cographs. Motivated by [1], we determine the convexity number for complementary prisms of disconnected graphs and of cographs, and we show lower bounds of $con(G\overline{G})$ when the diameter of G is at most two.

References

[1] M. C. Dourado, F. Protti, D. Rautenbach, and J. L. Szwarcfiter. *On the convexity number of graphs.* Graphs and Combinatorics, 28(3):333–345, 2012.

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Knot-Free Vertex Deletion Problem: Parameterized Complexity of a Deadlock Resolution Graph Problem

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Keywords: Knot; Deadlock resolution; FPT; W[1]-hard; ETH.

A *knot* in a directed graph G is strongly connected subgraph Q of G with size at least two, such that no vertex in $V(Q)$ is in-neighbor of a vertex in $V(G) \setminus V(Q)$. Knots are very important graph structure in the networked computation field, because they characterize deadlock occurrences into a classical distributed computation model, so-called OR-model. Given a directed graph G and a positive integer k, we present parameterized complexity analysis of the Knot-free Vertex Deletion (KFVD) problem, which consists of determining whether G has a subset $S \subseteq V(G)$ of size at most k such that $G[V \setminus S]$ contains no knot. KFVD is a graph problem with natural applications in deadlock resolution area, and it is close related to Directed Feedback Vertex Set. It is known that KFVD is NP-complete on planar graphs with bounded degree, but it polynomial time solvable on subcubic graphs [**?**].

In this paper we proof that: KFVD is W[1]-hard when parameterized by the size of the solution; it can be solved in $2^{k \log \varphi} n^{O(1)}$-time, but assuming SETH it cannot be solved in $(2-\epsilon)^{k \log \varphi} n^{O(1)}$-time, where φ is the size of the largest strongly connected subgraph of G; it can be solved in $2^{\phi} n^{O(1)}$-time, but assuming ETH it cannot be solved in $2^{O(\phi)} n^{O(1)}$-time, where ϕ is the number of vertices with out-degree at most k; unless $PH = \Sigma_p^3$, KFVS do not admit polynomial kernel even when $\varphi = 2$ and k is the parameter.

References

[1] Carneiro, A. D. A., Protti, F., Souza, U. S. (2017). Deadlock Graph Problems Based on Deadlock Resolution. In: Yixin Cao; Jianer Chen. International Computing and Combinatorics Conference. 1ed.: Springer, 2017, v. 1, p. 75-86.

A Parameterized Complexity Analysis of Clique and Independent Set in Complementary Prisms

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Keywords: Clique; Independent Set; Parameterized Complexity.

Introduced in the last decade by Haynes et al. [2], the complementary prism $G\bar{G}$ of the graph G is a variation of the well known prism. It arises from the disjoint union of the graph G and its complement $\bar{G}$ by adding the edges of a perfect matching joining pairs of corresponding vertices of G and $\bar{G}$. Despite being rather new graph class, classical graph properties such as domination, independence, cliques among others were studied and a polynomial time recognition algorithm was presented. Duarte et al. [1], have shown that given complementary prism $G\bar{G}$ and a integer k, it is NP-complete to decide whether $G\bar{G}$ has the following property: a clique of order k (denoted by k-Clique); a independent set of order k (denoted by k-Independent Set).

In this work, we studied the complexity of the k-Clique and k-Independent Set problems in complementary prisms from a parameterized complexity point of view. First, we proved that k-Clique and k-Independent Set have a kernel and therefore are Fixed-Parameter Tractable (FPT). Then, we showed that unless $NP \subseteq coNP/poly$, k-Clique and k-Independent Set do not admit polynomial kernel.

References

[1] M. A. Duarte, L. Penso, D. Rautenbach, and U. S. Souza. Complexity properties of complementary prisms. Journal of Combinatorial Optimization, 33(2):365372, 2017.

[2] T. W. Haynes, M. A. Henning, P. J. Slater, and L. C. van der Merwe. The complementary product of two graphs. Bulletin of the Institute of Combinatorics and its Applications, 51:2130, 2007.

The Diverse Vertex Covers Problem

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Keywords: Parameterized complexity, multiple solutions, FPT

One fundamental issue in applying algorithmic results in practice arises from the fact that the majority of computational problems are typically defined as *decision problems*, asking e.g. 'does the input contain structure X'? Practitioners are rarely satisfied with simple Yes/No answers to such questions and even a single solution X^* might not give enough insight into the instance at hand. Rather, one might want to find a '*small representative subset* of structures X in the input', a goal whose formulation may seem mathematically inconvenient. However, a set of solutions that are pairwise very similar does not provide much more insight than a single solution. In other words, there is a metric on the solution space of a problem such that in a representative subset of the solutions, the members should be pairwise far apart. Via the Vertex Cover problem, which is the parametric dual to clique, we show that such problems can admit efficient parameterized algorithms.

Formally, let δ be a metric on the space of all vertex covers of graphs. In the δ-Diverse Vertex Covers problem, given an n-vertex graph G, and three integers k, r, and d, we want to compute a set $\mathcal{S}$ of r distinct vertex covers of G of size at most k such that for each pair of distinct vertex covers S, $T \in \mathcal{S}$, $\delta(S,T) \geq d$. When $r = 1$, this problem corresponds to the NP-complete Vertex Cover problem. We show that if δ is the Hamming distance of the indicator vectors associated with the vertex covers, then the corresponding Hamming-Diverse Vertex Covers problem is fixed-parameter tractable when parameterized by $r + k$, i.e. solvable in time $f(r,k) \cdot n^c$ for some computable function f and constant c.

The δ-Diverse Cliques problem may be investigated using parametric duality: To obtain a clique in the input graph G, one computes a vertex cover X in the complement graph $\overline{G}$; $V(G) \setminus X$ is a clique in G.

Directed tree-width is FPT[4]

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Keywords: Directed tree-width, width parameters, FPT algorithms.

Width parameters in graphs are an estimation to how similar a graph is to a typical structure. Many hard problems can be efficiently solved on graphs with bounded width parameters by making use of classical algorithm construction approaches, like dynamic programming, exploiting the structure given by the width restrictions on the graph. In this work, we focus on the tree-width of directed graphs.

Robertson and Seymour showed that every undirected graph of tree-width at least $f(k)$ contains a $k \times k$ grid minor [2]. Approaches based on this result and on width parameters achieved great success in the design of algorithms for problems on undirected graphs. For directed graphs, an analogous definition for tree-width was proposed in [3] and a result analogous to the grid theorem was proved in [1].

By improving on a result from [3], we give an FPT algorithm, with parameter k, which decides whether a directed graph has tree-width at most $3k-2$ or admits a haven (as defined in [3]) of order k.

References

[1] K. Kawarabayashi and S. Kreutzer. “The Directed Grid Theorem”. In: *Proceedings of the Forty-seventh Annual ACM Symposium on Theory of Computing.* STOC '15. New York, NY, USA: ACM, 2015, pp. 655–664.

[2] N. Robertson and P.D. Seymour. “Graph minors V. Excluding a planar graph”. In: *Journal of combinatorial theory, series B* 41.01 (1986), pp. 92–114.

[3] T. Johnson et al. “Directed tree-width”. In: *Journal of combinatorial theory, series B* 82.01 (2001), pp. 138–154.

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P_3-Helly number of graphs with few P_4

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Keywords: P_3-convexity, Helly property, graphs with few P_4

Let $G = (V, E)$ be a graph. A set S is P_3-*convex* (resp. ${P^*}_3$-*convex*) if there is no vertex outside S having two neighbors in S (resp. there is no vertex outside S with two non-adjacent neighbors in S). A set of vertices $S \subseteq V$ is P_3-*Helly independent* (resp. ${P^*}_3$-*Helly independent*) if there is no vertex v such that $v \in \langle S \setminus \{w\} \rangle$ for every $w \in S$, where $\langle S \rangle$ is the smallest convex set containing S. The maximum cardinality of a P_3-Helly independent (resp. ${P^*}_3$-Helly independent) of a graph G is denoted by $h_{P_3}(G)$ (resp. $h_{{P^*}_3}(G)$).

There are several works related to P_3-convexities [1, 2]. The VP3HC (resp. VSP3HC) problem receives a graph G and an integer k and asks if $h_{P_3}(G) \geq k$ (resp. $h_{{P^*}_3}(G) \geq k$). The EP3HC and ESP3HC problems are the edge counterparts of the previous problems using the parameters h'_{P_3}, and $h'_{{P^*}_3}$. In this work we present bounds for h_{P_3}, $h_{{P^*}_3}$, h'_{P_3}, and $h'_{{P^*}_3}$ relating them to other well-known graph-theoretic parameters. Moreover, we show efficient solutions for these problems restricted to graph classes with few induced P_4.

References

[1] Araújo, R. T.; Sampaio, R. M.; Szwarcfiter, J. L; The convexity of induced paths of order three, *Eletron. Notes Discrete Math.*, 44, 109-114, 2013.

[2] Penso, L. D.; Protti, F.; Rautenbach, D.; Souza, U. S.; Complexity analysis of $P3$-convexity problems on bounded-degree and planar graphs, *Theor. Comput. Sci.*, 607, 83-95, 2015.

On the P_3-Hull Number for Strongly Regular Graphs

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Keywords: P_3 convexity, hull number, strongly regular

A set of vertices S of a graph G is P_3-convex if every vertex of G lies on a path with three vertices between two not necessarily distinct vertices in S. The P_3-convex hull of a set of vertices S is the smallest convex set containing S. The P_3-hull number $h(G)$ of a graph G is the smallest cardinality of a set of vertices whose convex hull is the vertex set of G. In this paper we establish some limits for P_3-hull number in the P_3 convexity for strongly regular graphs. A graph G is strongly regular if is k-regular and there are integers b and c such that every two adjacent vertices have b common neighbours and two non-adjacent vertices have c common neighbours. For disconnected strongly regular graph, if $\omega(G)$ is a number of connected components of G, $h(G) \leq 2.\omega(G)$, in connected graphs $h(G) \leq \left\lceil \frac{k}{1+b} \right\rceil + 1$ or alternatively $h(G) \leq \lceil \log_{c+1}(k.c + 1) \rceil + 1$.

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Sobre o número de Sierksma de um grafo

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Keywords: Convexidades, Convexidade geodética, número de Helly, número de Carathéodory e número de Sierksma

Convexidade em grafos e suas invariantes são tópicos bastante estudados na literatura, no entanto, há poucos trabalhos relacionados a invariante número de Sierksma na convexidade geodética, que é sobre caminhos mínimos em grafos. O presente trabalho realiza um estudo sobre esta convexidade com o objetivo de caracterizar e determinar limites superiores para o número de Sierksma de algumas classes de grafos, tais como grafos k-partidos e split. O número de Helly consiste no menor inteiro k, tal que todo conjunto $S \subseteq V$, com $k+1$ elementos, tem a propriedade $\bigcap_{u \in S} H(S \setminus \{u\}) \neq \emptyset$.

Um conjunto S de vértices de um grafo G é um conjunto de Carathéodory se o conjunto $\partial H(S)$ definido como $H(S) \setminus \bigcup_{u \in S} H(S \setminus \{u\})$ é não vazio. Já o número de Caratheodory é definido como a cardinalidade máxima de um conjunto de Carathéodory. Define-se o número de Sierksma como a cardinalidade do conjunto Sierksma independente máximo, onde S é Sierksma independente se existe $p \in S$ tal que $H(S \setminus \{u\}) \setminus \bigcup_{u \in (S \setminus \{p\})} H(S \setminus \{u\}) \neq \emptyset$.

As inequações de Sierksma [1] relacionam o número de Carathéodory, o número de Helly e o número de Sierksma: $e-1 \leq c \leq max\{h, e-1\}$. Sendo assim o nosso estudo foca nas classes de grafos onde o número de Helly é maior do que o número de Carathéodory.

References

[1] Marcel LJ van De Vel. *Theory of convex structures*, volume 50. Elsevier, 1993.

The Rank on the Graph Geodetic Convexity†

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Keywords: Graph Convexity · Geodetic Convexity · Rank

Several distributed computing models and information dissemination consider an initial set of "activated" nodes that spread some information over a network. In such models it is natural to think how the spread will be realized. In this way, many rules can be taken, where nodes are added to an initial activated node set. We consider a spread in a simple graph $G = (V, E)$ such that vertices are activated according to the distances among them. More specifically, given an initial active node set $S \subseteq V(G)$, a vertex w becomes active whenever there exists a pair $u, v \in S$ such that w belongs to a shortest path between u and v. We consider the problem of finding the size of a largest set such that no elements can be activated by the others. Formally, the geodetic convexity is that one in which a subset of vertices is *geodetically convex* if all the vertices of any shortest path between vertices of S belong to S. The *convex hull* $H(S)$ of S is the smallest convex set containing S. A set S is *convexly independent* if $v \notin H(S \setminus \{v\})$ for every $v \in S$. The *rank* $rk(G)$ of G is the largest size of a convexly independent set [1, 2]. In this work we give some simple lower bounds for the rank on the geodetic convexity and determine the exact value for complete k-partite graphs and powers of cycles. Moreover we present a polynomial-time dynamic programming algorithm for cacti.

References

[1] Ramos, I., Santos, V.F., and Szwarcfiter, J.L., *Complexity aspects of the computation of the rank of a graph*, Discrete Math. Theor. Comput. Sci., **16(2)**, (2014), 73–86.

[2] Kant, M.M., Sampaio, R.M., Santos, V.F., and Szwarcfiter, J.L., *On the geodetic rank of a graph*, J. Combin.., **8(2)**, (2017), 323–340.

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Covering a body using unequal spheres and the problem of finding covering holes

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Keywords: covering problem, clique, Gamma Knife radiosurgery

This work complements the development of a method to generate the covering of a solid body using spheres, as presented in the article "A new penalty/stochastic approach to an application of the covering problem: the Gamma Knife treatment". The article deals with coverings of convex bodies using unequal spheres S_i, $i \in N$, where N is an index set. One question was left to be answered later: how to ensure that there are no "holes" in the covering spheres structure?

For the matter of this presentation, it is assumed that the covering spheres structure has already been obtained and the objective is to certify that there are no "holes" in it. On the other way, it is under development the theoretical basis to prove that the generated covering structures don't present "holes".

Let G be the undirected graph $G(V, E)$ where V is the set of the centers of the spheres and E is the set of the edges, such that the edge $e_{ij} \in E$ if spheres S_i and S_j overlap each other. A method involving the geometrical properties of the cliques K_3 and K_4, as subgraphs of G, will be presented, which permits to identify the presence of "holes" in the covering structure.

New proposals for the Problem of Covering Solids using Spheres of Different Radii

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Keywords: Gamma Knife, Hybrid Methods, Integer Programming

Given a solid T, represented by a compact set in R^3, the aim of this work is to find a quasi-covering of T by a finite set of spheres of different radii. The volume occupied by the spheres on the outside of T is limited and some level of intersection between the spheres is allowed. The intersection is allowed because the application that motivated this study is the planning of a radio-surgery treatment, known as Gamma Knife. This problem can be formulated as a non-convex optimization problem with quadratic constraints and linear objective function. In this work, two linear integer mathematical formulations with binary variables are proposed for the Gamma Knife problem. Besides the formulations, a hybrid method is proposed as well. The hybrid method combines heuristic, data mining and an exact method. Computational results show that the proposed methods outperform the ones presented in the literature.

Connectivity of graphs of cubical polytopes

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Keywords: k-linked, cube, cubical polytope, graph of a polytope, connectivity, separator, linkedness.

A cubical d-dimensional polytope, or simply cubical d-polytope, is a d-polytope with all its *facets*, $(d-1)$-dimensional faces, being combinatorially equivalent to $(d-1)$-dimensional cubes. A graph of a polytope is the undirected graph formed by the vertices (0-dimensional faces) and the edges (1-dimensional faces) of the polytope.

The first part of the paper deals with the connectivity of graphs of cubical polytopes. We first establish that, for every dimension $d \geq 3$ and every integer $0 \leq \alpha \leq d-3$, the graph of a cubical d-polytope with minimum degree $d+\alpha$ is $(d+\alpha)$-vertex-connected. Secondly, we show that, for $d \geq 4$ and $0 \leq \alpha \leq d-3$, every vertex separator of cardinality $d+\alpha$ in the graph of a cubical d-polytope consists of all the neighbours of some vertex and breaks the polytope into exactly two components.

The second part of the paper deals with the stronger concept of linkedness. A graph with at least $2k$ vertices is *k-linked* if, for every set of $2k$ distinct vertices organised in arbitrary k pairs of vertices, there are k disjoint paths joining the vertices in the pairs. Larman and Mani in 1970 proved that the graphs of simplicial d-polytopes, d-polytopes with all its facets being $(d-1)$-simplices, are $(d+1)/2$-linked. This is the maximum possible linkedness, given the facts that a k-linked graph is at least $(2k-1)$-vertex-connected and that some of these graphs are d-vertex-connected but not $(d+1)$-vertex-connected. Here we establish that graphs of cubical d-polytopes are also $(d+1)/2$-linked, for every $d \neq 3$, which is again the maximum possible linkedness.

A strategy to select vertices as candidates for routers in a Steiner tree

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Keywords: algorithm, Steiner tree, heuristic, shortest paths

Let G be an undirected weighted graph with n vertices and m edges, and let W be a subset of vertices from G, a Steiner tree is a connected subgraph T from G that contains all vertices from W and its edge weights sum is minimum. Vertices in W are called terminals and all other vertices from G used to form T are called Steiner vertices. It is known that the decision version for the Steiner trees problem in graphs is NP-Complete, so proposals for new heuristics help the development of better approximation algorithms.

In this paper we present a heuristic algorithm based on the concepts of a good exact enumerative algorithm already proposed in the literature [DOP14]. The idea is to test the insertion of each non-terminal vertex as a router in a greedy way and check if it optimizes the actual solution tree. We do this until we reach the maximum numbers of routers $(k-2)$. Despite the many algorithms already developed for this problem, we present simple heuristics method and we show that the algorithm achieves good results in benchmark test bases for complete and sparse graphs with random weights edges.

References

[DOP14] Mitre Dourado, Rodolfo Oliveira, and Fábio Protti. Algorithmic aspects of steiner convexity and enumeration of steiner trees. *Annals of Operations Research*, 223(1):155–171, 2014.

Powers of Circular-Arc Models

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Keywords: circular-arc models, powers of models, multiplicative models

A *proper circular-arc (PCA)* model is a pair $\mathcal{M} = (C, \mathcal{A})$ such that C is a circle and $\mathcal{A}$ is a finite family of inclusion-free arcs of C. Each arc A of $\mathcal{A}$ has two *extremes*: its *beginning point* $s(A)$ and its *ending point* $t(A)$, which are the first and last points of A reached when C is clockwise traversed, respectively. A PCA model is a (c, ℓ)-CA model when the circumference of the circle is c and all arcs of $\mathcal{A}$ have length ℓ. In general, $\mathcal{M}$ is a *unit circular-arc (UCA)* model when it is a (c, ℓ)-CA model for some c, ℓ. Two PCA models are *equivalent* if the extremes of their arcs appear in the same order when C is clockwise traversed.

For any $A \in \mathcal{A}$, its *next* arc is defined as the arc $\mathrm{next}(A) = A'$ such that $s(A')$ is the last beginning point reached before $t(A)$ when C is clockwise traversed. The *k-th power* of A is defined recursively as $A^1 = A$ and $A^k = (s(A), t(\mathrm{next}(A^{k-1})))$, while the *k-th power* of a model $\mathcal{M}$ is $\mathcal{M}^k = (C, \{A^k \mid A \in \mathcal{A}\})$. For a (c, ℓ)-CA model $\mathcal{U}$, we define the *j-th multiple* of $\mathcal{U}$ as $j \cdot \mathcal{U} = (C, \{(s(A), s(A) + j\ell) \mid A \in \mathcal{A}\})$.

In this work we study the question of whether some model $\mathcal{M}$ is *k-multiplicative*, i.e., determining if the models $\mathcal{M}^i$ and $i \cdot \mathcal{M}$ are equivalent for all $i \leq k$. For $k = 1$, this question is precisely the recognition problem for UCA models, which was first solved by Tucker. Soulignac and Terlisky recently proposed a new characterization of (c, ℓ)-CA models that yields a simpler algorithm for the recognition of UCA models, based on Pirlot's synthetic graphs. In this work we generalize synthetic graphs to study the *multiplicative* problem. Our results can be applied to characterize which powers of UCA graphs are UCA graphs as well.

Digrafo de intersección de torneos transitivos maximales

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Keywords: torneos transitivos, digrafo de torneos transitivos

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Un *torneo* es un digrafo $T = (V, A)$ que posee exactamente una flecha para cada par de vértices. Un torneo T se dice *transitivo* si, para cualesquiera tres vérices $a, b, c \in V(T)$ se cumple la transitividad de la relación. Es decir, si $a \to b$ y $b \to c$ son flechas del digrafo, entonces $a \to c$ es una flecha del digrafo. Todo torneo transitivo tiene un vértice *fuente* y un vértice *sumidero*. Definimos el *digrafo de torneos transitivos* como el digrafo de intersección de subtorneos transitivos maximales $\tau(D)$ de D tal que

- $V(\tau(D))$ es el conjunto de todos los subtorneos transitivos maximales por contención del digrafo D y
- $A(\tau(D))$ es el conjunto de todas aquellas flechas definidas de la siguiente forma: si T_1 y T_2, son subtorneos transitivos maximales de D, f_1, f_2 y s_1, s_2 sus correspondientes vértices fuentes y sumideros, respectivamente, entonces $T_1 \to T_2$ si $s_1, f_2 \in T_1 \cap T_2$ y $f_1, s_2 \notin T_1 \cap T_2$.

Se presentan los resultados iniciales sobre el comportamiento de este operador y su vínculo con el operador clique y con el operador de líneas. Se analiza el operador en los digrafos de comparabilidad y además, la convergencia, divergencia y periodicidad en ciclos orientados con una cuerda.

Se estudia el comportamiento en orientaciones de un grafo. En particular, se consideran las distintas orientaciones del octaedro O_3, donde resulta que el operador converge en algunas y diverge en otras.

Extremal unit circular-arc models

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Keywords: minimal unit circular-arc models; extremal models

A *unit circular-arc* (UCA) model is a pair $\mathcal{M} = (C, \mathcal{A})$ such that C is a circle and $\mathcal{A}$ is a finite family of arcs of C, all having the same length. A UCA model $\mathcal{M}$ is called a (c, ℓ)-*CA* model when $|C| = c$ and the length of the arcs of $\mathcal{A}$ is ℓ. The model $\mathcal{M}$ is *equivalent* to a UCA model $\mathcal{M}'$ when the extremes of $\mathcal{M}$ and $\mathcal{M}'$ appear in the same order when their respective circles are clockwise traversed, whereas $\mathcal{M}$ and $\mathcal{M}'$ are *isomorphic* when their intersection graphs are isomorphic.

If $c \leq c'$ and $\ell \leq \ell'$ for every (c', ℓ')-CA model $\mathcal{M}'$ equivalent (resp. isomorphic) to $\mathcal{M}$, then $\mathcal{M}$ is said to be *minimal* (resp. *minimum*). It is already known that every UCA model is equivalent (resp. isomorphic) to some minimal (resp. minimum) UCA model. For $n \in \mathbb{N}$, each (c, ℓ)-CA model with maximum c (resp. ℓ) among those minimal models with n arcs are said to be *circle extremal* (resp. *arc extremal*). Similarly, the (c, ℓ)-CA models with maximum c (resp. ℓ) among those minimum models with n arcs are called *circle isoextremal* (resp. *arc isoextremal*). Finally, those models that are both circle and arc (iso)extremal simply are referred to as being *(iso)extremal*.

Lin and Szwarcfiter (Unit Circular-Arc Graph Representations and Feasible Circulations, SIAM J. Disc. Math., 22(1), pp. 409–423) left open the problem of determining the value c (resp. ℓ) of circle (resp. arc) isoextremal models. In this work we solve these problems and their analogous problems for extremal models, while we characterize those (circle, arc) (iso)extremal models, for every $n \in \mathbb{N}$.

Circular-arc Bigraphs and the Helly Subclass

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Keywords: Bicliques; Circular-arc bigraphs; Bipartite-helly; Helly.

A *bi-circular-arc model* is a triple $(C, \mathcal{I}, \mathcal{E})$ such that C is a circle, and $\mathcal{I}, \mathcal{E}$ are families of arcs over C. A bipartite graph $G = (V, W, E)$ is a *circular-arc bigraph* if and only if there exists a bi-circular-arc model $(C, \mathcal{I}, \mathcal{E})$ and a bipartition $a : V \cup W \to \mathcal{I} \cup \mathcal{E}$ such that, for every $v \in V, w \in W$, we have $a(v) \in \mathcal{I}$, $a(w) \in \mathcal{E}$, and $vw \in E$ precisely when $a(v) \cap a(w) \neq \emptyset$. In that case, we say that G *admits* the model $(C, \mathcal{I}, \mathcal{E})$.

We define the class of *Helly circular-arc bigraphs* based on the concept of bipartite-Helly families. A graph $G = (V, W, E)$ is said to be a Helly circular-arc bigraph if G admits a bi-circular-arc model $(C, \mathcal{I}, \mathcal{E})$ such that, for every biclique $K \subseteq V \cup W$, there exists a point $p \in C$ such that, for every $v \in K$, $p \in a(v)$. The class is trivially hereditary over induced subgraphs. We demonstrate that, if G is a C_6-free bipartite graph with no isolated vertices, G is a Helly circular-arc bigraph if and only if G^2 is a Helly circular-arc graph.

We study the recognition problem for the class of Helly circular-arc bigraphs. We prove that it is polynomial solvable for graphs without isolated vertices by presenting an algorithm that, given a bipartite graph G without isolated vertices, reduces the problem to the recognition of Helly circular-arc graphs over G^2 if G is C_6-free, and searches for forbidden subgraphs from a finite set if G has an induced C_6.

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Pursuit Games on graphs with few P4's

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Keywords: Pursuit Games, Cops and Robber, Meyniel Conjecture, Spy-Game, Primeval Decomposition

One of the most famous pursuit games on graphs is the Cops-and-Robber game. In this game, there are two players, $\mathcal{C}$ and $\mathcal{R}$, moving their pawns on vertices of a given graph G. Player $\mathcal{C}$ controls the cops while player $\mathcal{R}$ controls the robber. The objective of the cops is to capture the robber in a finite number of rounds. In this case, the cops win; otherwise, the robber wins.

Given a graph G, the copnumber of G, denoted by $c(G)$, is the minimum integer k such that $\mathcal{C}$ can win, independently of the robber's strategy, using exactly k cops in G. Kinnersley recently proved that, given an arbitrary graph G and an integer k, deciding if $c(G) \leq k$ is EXPTIME-complete [3].

Meyniel conjectured that for any connected graph G with n vertices, $c(G) = O(\sqrt{n})$ [4,2]. This conjecture remains unsolved. In fact, the soft Meyniel's conjecture, which states that, for any connected graph G with n vertices, $c(G) = O(n^{1-\varepsilon})$ for some $\varepsilon > 0$, also remains open.

In this work, we use the primeval decomposition technique to obtain polynomial time algorithms to compute the copnumber of $(q, q-4)$-graphs and P_4-tidy graphs. Furthermore, we prove that the Meyniel's conjecture is true for P_4-tidy graphs and $(q, q-4)$-graphs with at least q vertices.

We also use the primeval decomposition technique to obtain a polynomial time algorithm for $(q, q-4)$-graphs to the Spy Game, which is other pursuit game recently introduced by Nisse et al [1].

[1] N. Cohen, N. A. Martins F.M. Inerney, N. Nisse, S. Pérennes, R. Sampaio, Spy-game on graphs: Complexity and simple topologies, *Theoretical Computer Science*, (2018).
[2] P. Frankl, Cops and robbers in graphs with large girth and Cayley graphs, *Discrete Applied Mathematics*, **17** (1987), 301–305.
[3] W. B. Kinnersley, Cops and Robbers is EXPTIME-complete, *Journal of Combinatorial Theory Series B*, **111** (2015), 201–220.
[4] H. Meyniel, personal communication with P. Frankl, (1985).

Clobber game as executive functions test†

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Keywords: Combinatorial game; Clobber

The Clobber game was introduced by Albert et al. in 2002 [1]. This game is played on a graph G, with a black, or white, stone on each of its vertices (initial configuration). An initial configuration is k-reducible if it can be reduced to k stones after a succession of moves. A graph is strongly 1-reducible if, for any vertex v, any initial configuration that is not monochromatic outside v, can be reduced to one stone on v of either color. Theoretical results using Graph Theory were obtained by Dantas et al. [2] who proved that the cartesian product of two strongly 1-reducible connected graphs, and the r-power of a path with $r \geq 3$ are strongly 1-reducible. In this work, we introduce the use of the combinatorial game Clobber as an alternative to the traditional tests for executive functions (EFs), like Tower of Hanoi Test, and present experiments that evidence the use of Clobber game as a test to identify changes in EFs, especially if we scale up in order to identify the moment of performance stabilization.

References

[1] Albert, M.; Grossman, J.; Nowakowsky, R. and Wolfe, D. An Introduction to Clobber. Integers, Journal of Combinatorial Number Theory, 5(2), A1, pp. 1–12, 2005.

[2] Dantas, S.; Gravier, S. and Pará, T. Solitaire Clobber played on Cartesian product of graphs. Discrete Applied Mathematics, v. 182, pp. 84–90, 2015.

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On the Minimum Broadcast Time Problem

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Keywords: minimum broadcast time, bounded treewidth, split graphs, interval graphs

Let $G = (V, E)$ a simple, undirected graph, and $V_0 \subseteq V$ a subset of vertices called *originators*. A *k-step broadcast scheme* for G is a $(2k + 1)$ sequence $V_0, E_1, V_1, E_2, V_2, \dots, E_k, V_k = V$ such that every $1 \leq i \leq k$ verifies $V_i \subseteq V$, $E_i \subseteq E$ and the following three conditions hold: (1) each edge in E_i has exactly one endpoint in V_{i-1}, (2) no two edges in E_i share a common endpoint, and (3) $V_i = V_{i-1} \cup \{v : uv \in E_i\}$. The *Minimum Broadcast Time Problem* asks for the minimum integer number k such that a k-step broadcast scheme for G is possible. This NP-complete problem has been approached under different algorithmic techniques (see [Hromkovič et al., Dissemination of information in interconnection networks (broadcasting & gossiping), Combinatorial Network Theory (1996)] for a survey). However, few articles so far address the design of efficient algorithms for specific graph families (such as [Slater et al., Information dissemination in trees, SIAM Journal on Computing (1981)] or [A. Farley et al., Broadcasting in trees with multiple originators, SIAM Journal on Algebraic Discrete Methods (1981)]). In this work we explore the existence of polynomial time algorithms for split graphs and interval graphs with one originator, and for bounded treewidth graphs with an arbitrary number of originators.

On distance colorings, graph embedding and IP/CP models

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Keywords: algorithms, graph coloring, polyhedral combinatorics.

A simple graph $G = (V, E)$ is a non-empty finite set V of elements called vertices and a set E of non-ordered pairs of elements other than V, called edges. Each edge and of E is denoted by the pair of vertices $e = (v, w)$ that forms. In this case, the vertices v, w are the ends of the edge and, being said adjacent. An embedding of G into a surface is a drawing of the graph on the surface in such a way that its edges may intersect only at their endpoints. In the classic vertex coloring problem (VCP), a mapping $c : V \to \mathbb{N}$ such that $c(i) \neq c(j)$ for each $(i, j) \in E$. The maximum used color k, which is equivalent to the number of used colors in this problem, must be minimized. A generalization of VCP is the bandwidth coloring problem (BCP), where there is a distance function $d : E \to \mathbb{N}$ and the mapping must respect $|c(i) - c(j)| \geq d_{i,j}$ for each $(i, j) \in E$. When $d : E \to \{1\}$, the problem is reduced to VCP. In both, we want an embedding in R of the input weighted graph, that is, we want an assignment of positive integers to the vertices (colors) that respect the embedding process (whose length of a line segment is given by the weight of the edge between two adjacent vertices). Based on this point of view, in this work, we present an overview of our recent results involving these coloring problems with distance constraints, called by us **distance coloring problems**. In addition to feasibility and polynomiality for specific classes of graphs, will be emphasized our results in constraint programming (CP), integer programming (IP) and polyhedral combinatorics. We present the orientation and distance IP models, show that there is an equivalence between facet-defining inequalities between them.

On bicliques and the second clique graph of suspensions

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Keywords: graph theory, graph dynamics, clique graphs, bicliques.

The *clique graph* $K(G)$ of a graph G is the intersection graph of the set of all (maximal) cliques of G. The *second clique graph* $K^2(G)$ *of* G is defined as $K^2(G) = K(K(G))$. The main motivation for this work is to attempt to characterize the graphs G that maximize $|K^2(G)|$, as has been done for $|K(G)|$ by Moon and Moser in [1].

The *suspension* $S(G)$ of a graph G is the graph that results from adding two non-adjacent vertices to the graph G, that are adjacent to every vertex of G. Using a new *biclique* operator B that transforms a graph G into its *biclique graph* $B(G)$, we found the characterization $K^2(S(G)) \cong B(K(G))$. We also found a characterization of the graphs G, that maximize $|B(G)|$.

Here, a biclique (X, Y) of G is an ordered pair of subsets of vertices of G (not necessarily disjoint), such that every vertex $x \in X$ is adjacent or equal to every vertex $y \in Y$, and such that (X, Y) is maximal under component-wise inclusion. The biclique graph $B(G)$ of the graph G, is the graph whose vertices are the bicliques of G and two vertices (X, Y) and (X', Y') are adjacent, if and only if $X \cap X' \neq \varnothing$ or $Y \cap Y' \neq \varnothing$.

References

[1] J.W. Moon and L. Moser. *On cliques in graphs.* Israel J. Math. **3** (1965) 23–28.

On a Class of Proper 2-thin Graphs

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Keywords: interval graph, proper interval graph, proper thin, thin

The thinness and the proper thinness are recently introduced parameters of a graph [1,2]. A graph G is *k-thin* if there exists a k-partition and an ordering of $V(G)$ such that for all ordered triple (p, q, r) of vertices of $V(G)$, if p and q are in a same part and $(p, r) \in E(G)$, then $(q, r) \in E(G)$. Such an ordering is called *consistent*. A graph G is *proper k-thin* if there exists a k-partition and a consistent ordering of $V(G)$ such that for all ordered triple (p, q, r) of vertices of $V(G)$, if q and r are in a same part and $(p, r) \in E(G)$, then $(p, q) \in E(G)$. Such an ordering is called *strongly consistent*. The 1-thin graphs (resp. proper 1-thin graphs) are precisely the interval graphs (resp. proper interval graphs).

The complexity of recognizing whether a graph is k-thin or proper k-thin is open, even for fixed $k \geq 2$. In [2], it is shown that given a partition of $V(G)$, the problem of determining whether there exists a consistent (or strongly consistent) ordering of $V(G)$ is NP-complete, whereas given an ordering $<$ of $V(G)$, determining the minimum k-partitioning of $V(G)$ for which $<$ is consistent (or strongly consistent) can be done in polynomial time.

We introduce the subclass of *proper 2-thin of precedence* as those proper 2-thin graphs that admit a strongly consistent ordering with respect to the bipartition (X, Y) of $V(G)$ in which each vertex of X precedes each vertex of Y. We present a characterization and an algorithm to recognize this class in $O(n^2)$ time, among other results.

References:

[1] Mannino, C., Oriolo, G., Ricci, F., Chandran, S. (2007). The stable set problem and the thinness of a graph. *Operations Research Letters*, 35:1–9.
[2] Bonomo, F., Estrada, D. (2017). On the thinness and proper thinness of a graph. *CoRR*, abs/1704.00379.

On Orthodox Tree Representations of $\mathrm{K}_{n,m}$

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Keywords: Intersection graph, subtree graph, orthodox (h, s, t)-representation

A graph G has an (h, s, t)-representation if it can be represented as the intersection graph of subtrees of a tree T, such that the maximum degree of T is at most h, every subtree has maximum degree s and two vertices of G are adjacent if and only if their corresponding subtrees intersect in at least t vertices [1]. The class of graphs that admits an (h, s, t)-representation is denoted by $[h, s, t]$. An (h, s, t)-representation of G is *orthodox* if all leaves of a subtree are also leaves of T, and two vertices of G are adjacent if and only if their corresponding subtrees share a leaf [1]. The class of graphs that admits an orthodox (h, s, t)-representation is denoted by $\mathrm{ORTH}[h, s, t]$.

The class $[3, 3, t]$ and $\mathrm{ORTH}[3, 3, t]$ have been studied by Jamison and Mulder in [1,2]. In this work, we investigate under what conditions a graph $K_{m,n}$ belong to the class $\mathrm{ORTH}[3, 3, t]$. Three results are presented. We show that the graph $K_{5,5} \notin \mathrm{ORTH}[3, 3, 5]$, proving the conjecture by Jamison and Muder [2], for $t = 5$. We also show that the graph $K_{3,n} \in \mathrm{ORTH}[3, 3, t]$, for $t = 2\lceil \log_2 n \rceil + 3$. Finally, we present an algorithm that constructs an orthodox $(3, 3, t)$-representation of a graph $G \subseteq K_{n,n}$ for $n = 3^x$ with $x \in \mathbb{N}^*$, where $t = 2n + n/3 - 3$.

[1] JAMISON, R. and MULDER, H., Tolerance intersection graphs on binary trees with constant tolerance 3, Discrete Mathematics, vol. 215, n. 1 3, pp. 115 131, 2000.
[2] JAMISON, R. and MULDER, H., Constant tolerance intersection graphs of subtrees of a tree , Discrete Mathematics, vol. 290, n. 1, pp. 27 46, 2005.

Biclique Graphs of Interval Bigraphs and Circular-arc Bigraphs

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Keywords: Bicliques, Biclique graphs, Interval bigraphs, Circular-arc bigraphs, Co-comparability graphs

A *biclique graph* of a graph G is the intersection graph of all maximal bicliques in G. A *bipartite interval model* is a bipartition of a finite number of intervals on the real line. A *bipartite circular-arc model* is a bipartition of a finite number of arcs on a circle. An *interval bigraph* is the intersection graph of a bipartite interval model in which each vertex corresponds to an interval and two vertices share an edge if and only if both corresponding intervals intersect and do not belong to the same part. A *circular-arc bigraph* is the intersection graph of a bipartite circular-arc model in which each vertex corresponds to an arc and two vertices share an edge if and only if both corresponding arcs intersect and do not belong to the same part. Note that every interval bigraph is a circular-arc bigraph.

In this work, we present a sweepline algorithm for finding all maximal bicliques of an interval bigraph using a given bipartite interval model. A variation of this algorithm can be used for finding the maximal bicliques of a circular-arc bigraph.

We also present some structural properties of bicliques of an interval bigraph when viewed as a set of intervals of a bipartite interval model. We introduce the notion of *gaps* and *centers* of bicliques and, based on these structural properties, we show that all biclique graphs of interval bigraphs are $K_{1,4}$-free co-comparability graphs.

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B_1-EPG-Helly Graph Recognition

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Keywords: EPG; Intersection Graphs; Helly; NP-hard; Single Bend

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Edge-intersection graphs of Paths on a Grid (EPG) is a class that has been defined by Golumbic[1], in which vertices are represented as paths on a grid and two vertices are adjacent if and only if the corresponding paths intersect in at least an edge of the grid. If every path can be represented with at most k bends, then we say this graph has a $B_k{-}EPG$ representation.

A set of paths is edge-Helly when every subset of the paths that have pairwise edge intersections has at least one edge contained in their total intersection. A B_1-EPG representation is Helly if the family of path has the edge-Helly property.

In this work, we prove that the B_1-EPG-Helly graph recognition problem is NP-hard. The proof of the $NP-$hardness of B_1-EPG-Helly recognition involves a reduction from the One-In-Three 3SAT problem with positive variables, similar to the one used by Heldt et al. [2] for B_1-EPG.

The reduction encodes a 3SAT formula as a graph, in which some vertices correspond to the variables of the formula. The gadget used in the reduction forces any B_1-EPG Helly representation of the graph to have exactly one of the three variables in each clause using a horizontal segment to intersect with the gadget whereas the other two are forced to intersect the gadget vertically. The concepts of horizontal and vertical are defined relative to a specific verticex of the construction.

References

[1] Golumbic M., Lipshteyn M., Stern M.: Edge intersection graphs of single bend paths on a grid, Networks, 54, 2009.

[2] Heldt D., Knauer K., Ueckerdt T.: Edge-intersection graphs of grid paths: The bend-number, Discrete Applied Mathematics, 167, pp. 144-162, 2014.

The Terminal connection problem on strongly chordal graphs and cographs

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Keywords: terminal connection; Steiner tree; strongly chordal; cographs

A *connection tree* T of a graph G for a *terminal set* $W \subseteq V(G)$ is a tree subgraph of G such that leaves$(T) \subseteq W \subseteq V(T)$. Given a graph G and a terminal set W, the well-known Steiner tree problem in graphs consists in a optimization task whose goal is to find a connection tree of G for W which has the minimum possible number of vertices.

In this work, instead of looking for minimum order connection trees, we ask for the existence of connection trees which satisfy additional constraints. A non-terminal vertex of a connection tree T is called *linker* if its degree in T is exactly 2, and it is called *router* if its degree in T is at least 3. Given a graph G, a terminal set W and two non-negative integers ℓ and r, the Terminal connection problem (TCP) asks whether G admits a strict connection tree for W with at most ℓ linkers and at most r routers.

TCP was proved to be NP-complete even if r is bounded by a constant [2]. We extend such a result by showing that the problem remains NP-complete on strongly chordal graphs. An interesting by-product of our proof is that it separates the complexity of TCP from the complexity of Steiner tree, which is polynomial-time solvable on strongly chordal graphs [3]. In contrast, for the class of cographs, we prove that the complexity of TCP agrees with the complexity of Steiner tree, which is polynomial-time solvable [1].

References

[1] Colbourn, C., and Stewart, L.: Permutation graphs: connected domination and Steiner tree. *Discrete Mathematics* **86** (1990), 179–189.

[2] Dourado, C. M., Oliveira, A. R., Protti, F., and Souza, S. U.: Design of connection networks with bounded number of non-terminal vertices. Latin-American Workshop on Clique in Graphs 2012, *Matemática Contemporânea* **42** (2014), 39–48.

[3] White, K., Farber M., and Pulleyblank, W.: Steiner trees, connected domination and strongly chordal graphs. *Networks* **15** (1985), 109–124.

Complexity Analysis of the And/Or graph Solution Problem on Planar Graphs

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Keywords: And/or graph; Planar graph; Apex graph; NP-hardness

An and/or graph is an acyclic, edge-weighted directed graph containing a single source vertex such that every vertex v has a label $f(v) \in \{and, or\}$. A solution subgraph H of an *and/or*-graph must contain the source and obey the following rule: if an *and*-vertex (resp. *or*-vertex) is included in H then all (resp. one) of its out-edges must also be included in H[1]. While this problem is well studied[2,3], the case where the input graph is an planar graph remains unexplored. In this work, we provide a proof that the problem remains NP-hard even when restricted to either planar graphs or apex graphs having a single sink, we also provide a proof that this problem is FTP.

References

1. U. dos Santos Souza, F. Protti, and M. D. da Silva, Revisiting the complexity of and/or graph solution, *Journal of Computer and System Sciences*, vol. 79, no. 7, pp. 11561163, 2013.
2. S. Sahni, Computationally related problems, *SIAM Journal on Computing*, vol. 3, no. 4, pp. 262279, 1974.
3. U. dos Santos Souza and F. Protti, Tractability, hardness, and kernelization lower bound for and/or graph solution, *Discrete Applied Mathematics*, vol. 232, pp. 125 133, 2017.

Weighted proper orientations of trees and graphs of bounded treewidth

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Keywords: proper orientation number; weighted proper orientation number; minimum maximum indegree; trees; treewidth; parameterized complexity; W[1]*-hardness.*

Given a graph G, a weight function $w : E(G) \to \mathbb{N} \setminus \{0\}$, and an orientation D of G, we define $\mu^-(D) = \max_{v \in V(G)} w_D^-(v)$, where $w_D^-(v) = \sum_{u \in N_D^-(v)} w(uv)$. We say that D is a *weighted proper orientation* of G if $w_D^-(u) \neq w_D^-(v)$ whenever u and v are adjacent. We introduce the parameter *weighted proper orientation number* of G, denoted by $\overrightarrow{\chi}(G, w)$, which is the minimum, over all weighted proper orientations D of G, of $\mu^-(D)$. When all the weights are equal to 1, this parameter is equal to the *proper orientation number* of G, which has been object of recent studies and whose determination is NP-hard in general but polynomial-time solvable on trees. Here, we prove that the equivalent decision problem of the weighted proper orientation number (i.e., $\overrightarrow{\chi}(G, w) \leq k$?) is weakly NP-complete on trees but can be solved by a pseudo-polynomial algorithm whose running time depends on k. Furthermore, we present a dynamic programming algorithm to determine whether a general graph G on n vertices and treewidth at most $\mathbf{tw}$ satisfies $\overrightarrow{\chi}(G, w) \leq k$, running in time $\mathcal{O}(2^{\mathbf{tw}^2} \cdot k^{2\mathbf{tw}} \cdot n)$. We complement this result by showing that the problem is W[1]-hard on general graphs parameterized by the treewidth of G, even if the weights are polynomial in n.

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Sobre los grafos PVPG: una subclase de los grafos vértice intersección de caminos en una grilla

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Keywords: grafos de intersección, grafos VPG, Co-comparabilidad, subgrafos prohibidos

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Los grafos vértice intersección de caminos en una grilla (grafos VPG) son grafos cuyos vértices pueden ser representados como caminos en una grilla tal que dos vértices son adyacentes si y sólo si los caminos correspondientes comparten al menos un vértice de la grilla. Se sabe que reconocer a los grafos VPG es un problema NP-completo. En este trabajo, estudiamos la clase de los grafos PVPG, esta es una subclase de los grafos VPG tal que todos los caminos representantes están entre dos rectas paralelas y tienen sus puntos finales sobre estas rectas. Probamos que PVPG = Co-comparabilidad. Más aún, presentamos algunos subgrafos inducidos prohibidos minimales para la clase de los grafos B_1-PVPG (esto es, grafos PVPG donde cada camino tiene a lo sumo un bend).

Clique-divergence is not first-order expressible for the class of finite graphs

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Keywords: graph theory, graph dynamics, clique graphs, first-order logic, Ehrenfeucht-Fraïssé game.

The *clique graph*, $K(G)$, of a graph G is the intersection graph of its (maximal) cliques. The *iterated clique graphs* of G are then defined by: $K^0(G) = G$ and $K^n(G) = K(K^{n-1}(G))$. We say that G is *clique-divergent* if the set of orders of its iterated clique graphs, $\{|K^n(G)| : n \in \mathbb{N}\}$ is unbounded. Clique graphs and iterated clique graphs have been studied extensively, but no characterization for clique-divergence has been found so far.

Recently, it was proved that the clique-divergence is undecidable for the class of (not necessarily finite) automatic graphs, which implies that clique-divergence is not first-order expressible for the same class.

Here we strengthened the latter result by proving that the clique-divergence property is not first-order expressible even for the class of finite graphs. Logic expressibility has strong relations with complexity theory and consequently, new avenues of research are opened for clique graph theory.

On Clique-Inverse Graphs of Graphs with Bounded Clique Number

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Keywords: clique graph, clique-inverse graph.

The *clique graph* $K(G)$ of G is the intersection graph of the family of maximal cliques of G. For a family $\mathcal{F}$ of graphs, the family of *clique-inverse graphs of* $\mathcal{F}$ is defined as $K^{-1}(\mathcal{F}) = \{\mathcal{H} \mid \mathcal{K}(\mathcal{H}) \in \mathcal{F}\}$. Let $\mathcal{F}_p$ be the family of K_p-free graphs, that is, graphs with clique number at most $p-1$, for an integer constant $p \geq 2$. Deciding whether a graph H is a clique-inverse graph of $\mathcal{F}_p$ can be done in polynomial time; in addition, $K^{-1}(\mathcal{F}_p)$ can be characterized by a finite family of forbidden induced subgraphs for $p \in \{2, 3, 4\}$. In [1], the authors propose to extend such characterizations to higher values of p. A natural conjecture that then arises is: Is there a characterization of $K^{-1}(\mathcal{F}_p)$ by means of a *finite* family of forbidden induced subgraphs, for any $p \geq 5$? In this work we show that this conjecture is true.

References

[1] Protti, F. and Szwarcfiter, J. L. Clique-Inverse Graphs of K_3-free and K_4-free Graphs. Journal of Graph Theory 35 (2000) 257–272.

The unit-demand envy-free princing problem applied to the sports entertainment industry

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Keywords: game theory, integer nonlinear programming, matching, maximum profit, ticket sales.

In this work, we discuss on the feasibility of applying the unit-demand envy-free pricing problem as strategy in modeling the sale of tickets for sporting events. An mathematical formulation in integer nonlinear programming of the problem is proposed, where assumes that the products have different price ranges, that the valuations of the products consumers are given by reserving prices, that consumers are grouped into segments, that the supply of each product is limited, that consumers can claim several units of the same product and also that there is no price difference for the same product. The goal is to maximize the seller's revenue, but in a way that buyer satisfaction. Preliminary computational experiments demonstrate the adequacy of the proposed formulation.

Worst cases in constrained LIFO pick-up and delivery problems

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Keywords: Pick-up and delivery, Permutation Graphs, Bounded Chromatic Number

Pick-up and delivery operations in transportation tasks are usually performed using stacks. The Last In First Out nature of stacks imposes restrictions to the order in which pick-ups and deliveries are performed. In this work, we consider both, limited and unlimited capacity stacks and we study the worst cases on the minimum number of stacks needed to perform any given sequences of pick-up and delivery operations allowing simple modifications on the delivery route. These problems amount to computing the smallest bounded chromatic number in a family of permutation graphs.

By allowing the reversion (R) of the delivery route one can save from one third (capacity 2) to half (unlimited capacity) on the number of stacks needed to perform any pair of pick-up and delivery routes in comparison with the worst case in which no modification is allowed. By allowing the modification of the starting point (S) one can always save half of the stacks. By allowing both types of modifications (R + S) one can always save at least half of the stacks (a few more in case of unlimited stacks) and at most one less than 3 quarters of the stacks.

The following table summarizes the results obtained in this work for n pairs of pick-up and delivery operations. Each row considers a different set of allowed operations and each column is related to a different stack capacity.

Cap.	∞		h		2	
	LB	UB	LB	UB	LB	UB
R	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$h \times \lfloor \frac{n}{2h-1} \rfloor$	$\lfloor \frac{\lfloor \frac{n-1}{h} \rfloor + n + 1}{2} \rfloor$	$\lfloor \frac{2n}{3} \rfloor$	$\lceil \frac{2n}{3} \rceil$
S	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$
R+S	$\lfloor \frac{n}{4} \rfloor + 1$	$\lceil \frac{n}{2} \rceil - \lfloor \frac{n}{22} - \frac{12}{11} \rfloor$	$\max(\lfloor \frac{n}{4} \rfloor + 1, \lceil \frac{n}{h} \rceil)$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$

Size Multipartite Ramsey Number

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Keywords: Generalized Ramsey number, Complete graph, Multipartite graph

A great challenge in graph theory is the determination of the celebrated *Ramsey numbers* $r(m, n)$, the smallest r such that every red-blue coloring of the edges of the clique K_r yields either a red copy of K_m or a blue copy of K_n.

Consider the following generalization. Let $K_{c\times s}$ denote the complete multipartite graph having c classes with s vertices per each class. Given an integer $c \geq 2$ and graphs $G_1, \ldots, G_k$, the *size multipartite Ramsey number* $m_c(G_1, \ldots, G_k)$ denotes the smallest positive integer s (if it exists) such that any k-coloring of the edges of $K_{c\times s}$ contains a monochromatic copy of G_i in color i for some i, $1 \leq i \leq k$. Particularly interesting, the number $m_2(G_1, \ldots, G_k)$ produces the widely studied *bipartite Ramsey number* $b(G_1, \ldots, G_k)$.

Burger, Grobler, Stipp, and van Vuuren [Discr. Math. 283 (2004) 45–49 and Utilitas Math. 66 (2004) 137–163] introduced and investigated the size multipartite Ramsey numbers $m_c(G_1, G_2)$ where each G_i is a complete multipartite graph. Since then, these numbers have been studied for special classes of graphs and several colors.

In this work, we discuss the numbers $m_c(G_1, \ldots, G_k)$ when each G_i is a bipartite graph K_{n,n_i} for a fixed positive integer n. We show some upper bounds using density arguments and some lower bounds using connections between classical Ramsey numbers and some concepts closely related to combinatorial designs (Hadamard matrix, strongly regular graph). These bounds are achieved for some instances, which allow us to obtain some exact classes. It is worth mentioning that exact values of Ramsey numbers are very often highly non-trivial to establish.

Maximum number of edges in graphs with prescribed maximum degree and matching number

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Keywords: maximum number of edges, graph classes, block duplicate graphs.

An extremal problem that dates back to 1960 is to determine the maximum number of edges that a graph can have if its maximum degree, denoted by $\Delta(G)$, and matching number are bounded. A graph satisfying the mentioned properties with the maximum number of edges is called an edge-extremal graph. This problem was solved [1] for general graphs and, depending on the parity of $\Delta(G)$, the edge-extremal graphs are a disjoint union of cliques and stars of a given size or a disjoint union of copies of a special graph H and stars. Of note is that the graph H is not chordal.

A natural problem that arises is to investigate how the maximum number of edges changes if we restrict the structure of the graphs considered. For instance, if we require the graphs to belong to a given graph class. In this work, we determine the number of edges of the edge-extremal graphs in the class of block duplicate graphs [2], a subclass of chordal graphs. We show that a disjoint union of cliques and stars of a given size is an edge-extremal graph in this class and conjecture that this is also the case for chordal graphs. We also determine the number of edges of the edge-extremal graphs in a graph class $\mathcal{C}$, when $\mathcal{C}$ is such that its graphs admit a proper edge-coloring with $\Delta(G)$ colors. Moreover, if the star with $\Delta(G)$ leaves belongs to $\mathcal{C}$, then a disjoint union of such stars is an edge-extremal graph in $\mathcal{C}$. Examples of graph classes satisfying this condition are bipartite graphs, outerplanar graphs with maximum degree at least 3 and planar graphs with maximum degree at least 7.

[1] V. Chvátal and D. Hanson, Degrees and matchings. *J. Combin. Theory Ser. B*, 20:128–138, 1976.
[2] M. C. Golumbic and U. N. Peled, Block duplicate graphs and a hierarchy of chordal graphs. *Discrete Appl. Math.*, 124:67–71, 2002.

An extremal problem on the interval counts

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Keywords: extremal problem, interval count, split graph, trivially perfect graph

The *interval graph* of a family $\mathcal{R}$ of closed intervals of the real line is the graph G such that $V(G) = \mathcal{R}$ and, for all distinct $I, J \in V(G)$, $(I, J) \in E(G)$ if and only if $I \cap J \neq \emptyset$. We call $\mathcal{R}$ an *interval model* of G. An *order* $P = (X, \prec)$ is a binary relation $\prec$ on the set X which is irreflexive and transitive. Moreover, P is an *interval order* if there is an interval model $\mathcal{R} = \{[\ell_x, r_x] \mid x \in X\}$ such that $x \prec y$ if and only if $r_x < \ell_y$. The minimum number $IC(G)$ (resp. $IC(P)$) of distinct lengths of intervals required in a model of G (resp. P) is called *interval count* of G (resp. P). An interval graph G is *trivially perfect* (in class TP) if G is P_4-free. An interval graph G is a *split* graph (in class Split) if there exists a partition (K, I) of $V(G)$ in which K is a clique and I is an independent set. Fishburn [1] introduced the extremal problem of determining

$$\sigma_{\mathcal{C}}(k) = \min\{|X| \mid P = (X, \prec) \text{ is in class } \mathcal{C} \text{ and } IC(P) \geq k\}$$

and $\tilde{\sigma}_{\mathcal{C}}(k)$ which is defined analogously for graphs. Fishburn conjectured that $\sigma_{\mathcal{C}}(k) = 3k-2$ for $\mathcal{C}$ as the class of general interval orders, having proved the conjecture for all $k \leq 7$. However, no results on $\tilde{\sigma}_{\mathcal{C}}(k)$ are known. We study $\sigma_{\mathcal{C}}(k)$ and $\tilde{\sigma}_{\mathcal{C}}(k)$ for $\mathcal{C} \in \{\text{TP}, \text{Split}\}$. Among other results, we prove that $\sigma_{TP}(k) = 3k-2$, $\sigma_{\text{SPLIT}}(k) = 3k-2$, $\tilde{\sigma}_{TP}(k) = \frac{(3^k-1)}{2}$, $\tilde{\sigma}_{\text{SPLIT}}(k) = 3k-1$.

Fishburn's conjecture for general interval orders and graphs remains open.

References:

[1] P. C. Fishburn. *Interval Orders and Interval Graphs*, John Wiley & Sons, 1985.