Minimizing the Solid Angle Sum of Orthogonal Polyhedra and Edge Guarding

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A well known problem in Art Galleries and illumination problems, is that any orthogonal polygon can always be guarded using $\lfloor \frac{n}{4} \rfloor$ guards. In this talk we extend these results to edge guarding of orthogonal polyhedron in $\mathbb{R}^3$.

To obtain our results, we generalize generalize to $\mathbb{R}^3$ the well-known result that in an orthogonal polygon with n vertices, $(n+4)/2$ of them are convex and $(n-4)/2$ of them are reflex. We define a vertex of a polyhedron to be convex on the faces if it is convex or straight in all the faces where it participates, and to be reflex on the faces otherwise. If a polyhedron with n vertices and genus g has k vertices of degree greater than 3 (in its 1-skeleton), we prove that it has $(n+8-8g+3k)/2$ vertices that are convex on the faces and $(n-8+8g-3k)/2$ vertices that are reflex on the faces.

We also give a characterization for the orthogonal polyhedron in $\mathbb{R}^3$ that minimize the sum of its internal solid angles, and prove that their minimum angle sum is $(n-4)\pi$ and their maximum angle sum is $(3n-24)\pi$.

If time allows, we will prove that if the orthogonal polyhedron has k_4 vertices of degree 4, k_6 vertices of degree 6, genus g and h_m holes on its faces, then we can guard it using at most $(11e - k_4 - 3k_6 - 12g - 24h_m + 12)/72$ $\frac{\pi}{2}$-edge guards (i.e., having a visibility angle of $\pi/2$ in the relative interior of each edge).

References

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