

# Book of Abstracts

## Latin American Workshop on Cliques in Graphs Discrete Mathematics and Applications Workshop

Rio de Janeiro, Brazil — November 25th, 2020





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Latin American Workshop on Cliques in Graphs  
Discrete Mathematics and Applications Workshop

Rio de Janeiro, Brazil — November 25th, 2020

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# Preface

Brazil, July 22nd, 2020. Four months had passed of social isolation. The Coronavirus Pandemic had spread throughout the world, and the 9th edition of the Latin American Workshop on Cliques in Graphs, which would take place in Chile, had been canceled. It was within this scenario that Celina M. H. de Figueiredo and Márcia Cappelle, members of the LAWCG steering committee, decided to hold the event online, together with the Discrete Mathematics and Applications (MDA) Workshop, organized annually by Simone Dantas and Telma Pará. The joint event was less than four months away, but the two traditional workshops should not be interrupted.

LAWCG ([www.lawcg.mat.br](http://www.lawcg.mat.br)) is meant to foster interaction within the Latin American Graph Theory and Combinatorics community, whose research interests include cliques, clique graphs, the behavior of cliques and other topics in Graph Theory. Previous editions of the workshop were held in Rio de Janeiro, Brazil (2002), La Plata, Argentina (2006), Guanajuato, Mexico (2008), Itaipava, Brazil (2010), Buenos Aires, Argentina (2012), Pirenópolis, Brazil (2014), La Plata, Argentina (2016), and Rio de Janeiro, Brazil (2018).

MDA ([www.mda.uff.br](http://www.mda.uff.br)) is a national scientific event which brings together researchers from renowned research centers in the field of Discrete and Combinatorial Mathematics, and students and teachers from public high schools. Since 2013, MDA has promoted the dissemination of Mathematics through activities inserted in projects supported by Brazilian funding agencies CAPES, CNPq and FAPERJ.

We are grateful to the invited speakers João Meidanis (UNICAMP, Brazil), Carmen Ortiz (UV, Chile) and Mónica Villanueva (USACH, Chile) for promptly accepting our invitation for this year's event. The scientific community also supported with enthusiasm the LAWCG + MDA'20 initiative with 58 poster submissions by 126 authors from eight countries: Argentina, Brazil, Chile, France, Mexico, The Netherlands, Norway, and Portugal.

My sincere thanks to the program and the organizing committees that worked together as a single group. The team was composed by nine professors and researchers who collaborated in each step of the process in an exceptional way, so that this event could take place. The members, 80% of which are women, represent four Brazilian regions: Northeast, Midwest, South and Southeast. In addition to the challenge of organizing, in a short period of time, an online event with an innovative structure through online meetings, this edition also leaves as legacy the LAWCG permanent link website, that gathers all documents from previous events.

Finally, LAWCG + MDA'20 has the honor of celebrating the 70th birthday of professor and researcher Célia Picinin de Mello who, in addition of being a very dear and special person, contributed significantly to the area of Computer Science, both in training human resources and in the production of important scientific results. With our gratitude and love, we hope that our friend, colleague and professor enjoys this tribute.

November 25th, 2020  
Rio de Janeiro, Brazil

Simone Dantas  
(General Chair)

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# Conference Program

|  |   |
|--|---|
| <br>(GMT-3) | November 25th, Thursday   |
| 14:40  | <b>Opening Time</b>   |
| 14:50  | <b>Inauguration Ceremony</b><br>LAWCG Steering Committee speech<br>MDA Steering Committee speech<br>Program Committee speech                        |
| 15:20  | <b>Network Science: a primer</b><br>chair: Sulamita Klein<br>speaker: João Meidanis, UNICAMP, Brazil  |
| 16:10  | <b>Best poster award winners</b>  |
| 16:30  | <b>A Survey on Independent Sets in Graphs</b><br>chair: Jayme Szwarcfiter<br>speakers: Carmen Ortiz, UV, Chile, and Mónica Villanueva, USACH, Chile |
| 17:20  | <b>Closing Ceremony</b>   |





# Abstracts of Talks

## Network Science: a primer

João Meidanis  
UNICAMP, Brazil

Nov 25th  
3:20pm

**Keywords:** *graph-like structures; Network Science; scale-free networks*

Euler's work on the Bridges of Königsberg problem is usually regarded as the birth of Graph Theory. But combinatorialists were not alone studying graph-like structures. In the beginning of the 20th century, sociologists started studying social networks. The idea of "six degrees of separation" (any human on Earth can reach any other by at most six acquaintance links) is from this period. More recently, physicists took to looking at real networks focusing on degree distributions, hubs, communities, preferential growth and other evolution hypotheses, and the like. In this lecture we will review the main points of modern Network Science, exploring scale-free networks, degree correlations, robustness and its relationship to spreading phenomena, e.g., pandemic modeling.

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Nov 25th  
4:30pm

## A Survey on Independent Sets in Graphs

Carmen Ortiz<sup>1</sup> and Mónica Villanueva<sup>2</sup>

<sup>1</sup>UV, Chile; <sup>2</sup>USACH, Chile

**Keywords:** *maximal independent set; independent graph; counting; enumerating*

An *independent set* of  $G = (V, E)$  is a subset  $S$  of  $V$  such that no two vertices are adjacent.  $S \subseteq V$  is a *maximal independent set* (mis for short) if it is not properly contained in any other independent set of  $G$ . The family of mis of  $G$  is denoted by  $M(G)$  and its cardinality is  $\mu(G)$ . Prodinger and Tichy (1982) introduced the *Fibonacci number*  $f(G)$  of a graph  $G$  as the number of independent sets of  $G$ , not necessarily maximal, including the empty set.

Valiant (1979) showed that the problem of counting the number of mis is  $\#\mathcal{P}$  – *complete* for a general graph. Füredi (1987) determined a bound for  $\mu(G)$  when  $G$  is a connected graph with at least 50 vertices. Independently, Griggs et al. (1988) established the same bound and characterized the extremal graphs that attain this bound for graphs with six or more vertices. This problem has been studied for various graph classes, including trees (Wilf (1986)), graphs with at most one cycle (Jou and Chang (1997)), graphs with  $r$  cycles (Sagan and Vatter (2006) and Goh et al. (2006)), bipartite graphs (Liu (1993)), triangle-free graphs (Hujter and Tuza (1993) and Chang and Jou (1999)) and unicyclic graphs (Koh et al. (2008) and Pedersen and Vestergaard (2005)). Euler (2005) calculated  $\mu(G)$  when  $G$  is a grid graph with up to five rows.

The problem of enumerating all maximal independent sets of a graph has also been considered by different authors. For a general connected graphs Tsukiyama et al. (1977) developed the best algorithm. Leung (1984) enumerated all mis of interval graphs, chordal graphs and circular arc graphs. Yu and Chen (1993) solved the problem for permutation graphs. Hota et al. (1999) did the same for trapezoid graphs.

The intersection graph  $\mathcal{I}(G)$  of the family of all maximal independent sets of a graph  $G$  is called the *Independent Graph* of  $G$ . Each vertex of  $\mathcal{I}(G)$  corresponds to a mis of  $G$  and two vertices are adjacent in  $\mathcal{I}(G)$  if their corresponding mis have non-empty intersection in  $G$ . Analogously, the *Clique Graph*  $\mathcal{K}(G)$  is the intersection graph of all cliques of  $G$ . Ortiz and Villanueva (2012) determined  $\mu(G)$  and generated the family of mis in a caterpillar graph and characterized the independent graph. Ortiz and Villanueva (2017) did the same for grid graphs with two rows.

In this work we survey results on these problems related to independent sets: counting independent sets and enumerating maximal independent sets.

# Abstracts of Posters

## Total coloring of some unitary Cayley graphs

D. Castonguay<sup>1</sup>, C.M.H. de Figueiredo<sup>2</sup>, L.A.B. Kowada<sup>3</sup>, C.S.R. Patrão<sup>2,4</sup>, D. Sasaki<sup>5</sup> and M. Valencia-Pabon<sup>6</sup>

<sup>1</sup>INF - Universidade Federal de Goiás, Brazil; <sup>2</sup>COPPE - Universidade Federal do Rio de Janeiro, Brazil; <sup>3</sup>IC - Universidade Federal Fluminense, Brazil; <sup>4</sup>Instituto Federal de Goiás, Brazil; <sup>5</sup>IME - Universidade Estadual do Rio de Janeiro, Brazil; <sup>6</sup>LIPN, Université Sorbonne Paris Nord, France

**Keywords:** *unitary Cayley graph; total coloring; coloring*

A total coloring of a graph  $G$  is an assignment of colors to the elements (vertices and edges) of  $G$  so that adjacent elements have different colors. The total chromatic number is the least number of colors needed for a total coloring. The well known Total Coloring Conjecture (TCC) states that the total chromatic number of a graph  $G$  is either  $\Delta(G) + 1$  or  $\Delta(G) + 2$ . In this work, we exhibit two new families of unitary Cayley graphs where all graphs have a total coloring with  $\Delta(G) + 1$ .

---

## Paths On Hosts: $B_1$ -EPG, EPT and VPT Graphs

Liliana Alcón<sup>1,2</sup>, María Pía Mazzoleni<sup>1</sup> and Tanilson Santos<sup>3,4</sup>

<sup>1</sup>Universidad Nacional de La Plata, Argentina; <sup>2</sup>CONICET - Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina; <sup>3</sup>Universidade Federal do Tocantins, Brazil; <sup>4</sup>Universidade Federal do Rio de Janeiro, Brazil

**Keywords:**  *$B_1$ -EPG graphs; Helly property; EPT; VPT*

This research contains as a main result the prove that every Chordal  $B_1$ -EPG graph is simultaneously in the graph classes VPT and EPT. In addition, we describe a set of graphs that define Helly- $B_1$ -EPG families. In particular, this work presents some features of non-trivial families of graphs properly contained in Helly- $B_1$  EPG, namely Bipartite, Block, Cactus and Line of Bipartite graphs.

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## A Matheuristic Approach for the Weighted Just-in-Time Scheduling Problem

Rainer Amorim<sup>1</sup>, Rosiane de Freitas<sup>2</sup>, Arthur Pessoa<sup>3</sup> and Eduardo Uchoa<sup>3</sup>

<sup>1</sup>Universidade Federal do Amazonas - Instituto de Ciências Exatas e Tecnologia, Brazil;

<sup>2</sup>Universidade Federal do Amazonas - Instituto de Computação, Brazil; <sup>3</sup>Universidade Federal Fluminense, Brazil

**Keywords:** *identical parallel machine; arc time; earliness tardiness scheduling; tardiness scheduling problem; hash table*

This work presents a hybrid exact-heuristic algorithmic approach, based on an arc-time indexed mixed-integer programming formulation and a generalized evolutionary based on a strong local search, to better solve classical parallel machine scheduling problems involving weighted tardiness and weighted earliness-tardiness penalties, with independent jobs and arbitrary processing times. Thus, the proposed method of this research is based on two steps. In the first step, some selected arcs from local optimal solutions, generated by a Genetic Algorithm based on a strong Local Search with generalized pairwise interchanges (GLS), are given as input to an integer programming formulation. In the second step, in order to produce improved solutions, the exact-full IP formulation is solved at CPLEX with the selected arcs from GLS. The computational experiments present competitive results when compared with the previous best results of the literature, including large instances up to 500 jobs on 2-10 identical parallel machines.

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## Layout de Redes de Sensores Sem Fio com Múltiplas Origens e Destinos: Uma Abordagem Combinatória

Igor Morais, Liss Faulhaber, Felipe Henriques and Pedro Henrique González

Centro Federal de Educação Tecnológica Celso Suckow da Fonseca (CEFET/RJ), Brazil

**Keywords:** *redes de sensores sem fio; programação inteira; otimização combinatória*

This paper presents an exact method for the Wireless Sensor Network Planning Problem with Multiple Sources/Destinations (WSNDP-MOD). In this work, we want to minimize the number of sensors in the network topology in a given region of interest, in order to balance data traffic between multiple sources and destinations. To solve the proposed problem, we will define an Integer Programming model and an auxiliary graph that will be used with the model. Computational experiments were performed using a real topology.

## Logarithmic-Sobolev and Poincaré inequalities for the simple random walk on the Hanoi graph

Rodrigo Marinho

Universidade de Lisboa, Portugal

**Keywords:** *algebraic connectivity; log-Sobolev constant; spectral gap; tower of Hanoi*

We exhibit a way to obtain lower bounds on the algebraic connectivity and on the log-Sobolev constant of a large class of graphs, for instance discrete approximations of fractals. We illustrate the method obtaining upper bounds on the inverse of both parameters on the Hanoi graphs. Our results imply upper bounds on the time that the random walk on the configurations of the tower of Hanoi puzzle, with 3 towers and  $m$  disks, gets lost.

---

## On total coloring of circulant graphs

Matheus Adauto<sup>1</sup>, Sergio Fusquino<sup>2</sup>, Mauro Nigro<sup>2</sup> and Diana Sasaki<sup>2</sup>

<sup>1</sup>Federal University of Rio de Janeiro, Brazil; <sup>2</sup>Rio de Janeiro State University, Brazil

**Keywords:** *total coloring; circulant graphs; regular graphs*

A  $k$ -total coloring of a graph  $G$  is an assignment of  $k$  colors to the elements of  $G$  such that adjacent elements have different colors. The total chromatic number  $\chi''(G)$  is the smallest integer  $k$  for which  $G$  has a  $k$ -total coloring. Clearly,  $\chi''(G) \geq \Delta(G) + 1$ , and the Total Coloring Conjecture (TCC) states that for any simple graph  $G$ , the total chromatic number is either  $\Delta(G) + 1$  (graphs are called Type 1) or  $\Delta(G) + 2$  (graphs are called Type 2). In this work, we determine the total chromatic number of all members of an infinite family of 4-regular circulant graphs.

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## Equitable total coloring of Blowup snarks

Isabel Gonçalves<sup>1</sup>, Simone Dantas<sup>1</sup> and Diana Sasaki<sup>2</sup>

<sup>1</sup>Fluminense Federal University, Brazil; <sup>2</sup>Rio de Janeiro State University, Brazil

**Keywords:** *k-total coloring; total coloring; snark graph; blowup snark graph*

The search for counter examples to the Four Color Conjecture originated snarks, which are bridgeless cubic graphs with chromatic index 4. A total coloring of a graph  $G$  is an association of colors to the vertices and edges of  $G$ , so that adjacent or incident elements do not have the same colors. If the difference between cardinalities of any two colors is at most one, the coloring is said equitable. In this work we construct an equitable total coloring for the infinite family of snarks named Blowups.

## The Strict Terminal Connection Problem on Chordal Bipartite Graphs

Alexsander Melo<sup>1</sup>, Celina Figueiredo<sup>1</sup> and Uéverton Souza<sup>2</sup>

<sup>1</sup>Federal University of Rio de Janeiro, Rio de Janeiro, Brazil; <sup>2</sup>Federal Fluminense University, Niterói, Brazil

**Keywords:** *terminal connection; strict connection tree; chordal bipartite graphs; NP-completeness*

A strict connection tree of a graph  $G$  for a non-empty subset  $W$  of  $V(G)$ , called terminal set, is a tree subgraph of  $G$  whose leaf set is equal to  $W$ . A non-terminal vertex of a strict connection tree  $T$  is called linker if its degree in  $T$  is exactly 2, and it is called router if its degree in  $T$  is at least 3. The Strict Terminal Connection problem (S-TCP) has as input a graph  $G$ , a non-empty subset  $W$  of  $V(G)$  and two non-negative integers, and it asks whether  $G$  admits a strict connection tree for  $W$  with at most  $l$  linkers and at most  $r$  routers. In this work, we prove that S-TCP is NP-complete even if  $l$  is fixed and the input graph is restricted to chordal bipartite graphs.

## Bounds for Range-Relaxed Graceful game

Deise de Oliveira<sup>1</sup>, Simone Dantas<sup>1</sup> and Atilio Luiz<sup>2</sup>

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**Keywords:** *graceful game; range relaxed graceful game; graph labeling*

Tuza (2017) contributed to the area of graph labeling presenting many results in his seminal paper and proposing new labeling games. We investigate the Range-Relaxed Graceful game (RRG game) and present a lower bound for the number of available labels for which Alice has a winning strategy in the RRG game on a simple graph, on a cycle and on a path graph.

## The sandpile groups of outerplanar graphs

Carlos Alejandro Alfaro<sup>1</sup> and Ralihe Raul Villagrán<sup>2</sup>

<sup>1</sup>Banco de México, México; <sup>2</sup>CINVESTAV-IPN, México

**Keywords:** *sandpile group; outerplanar graph; Smith normal form*

We compute the sandpile groups of outerplanar graphs. The method can be used to determine the algebraic structure of the sandpile groups of other planar graph families.

## Properties of fullerene graphs with icosahedral symmetry

Thiago M. D. Silva<sup>1</sup>, Diego Nicodemos<sup>2</sup> and Simone Dantas<sup>3</sup>

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**Keywords:** *graph theory; Fullerene graphs; icosahedral symmetry*

Fullerene graphs have become a popular class of graphs recently. They are characterized as 3-regular and 3-connected planar graphs, with only pentagonal or hexagonal faces. The fullerene graph with icosahedral symmetry is a particular class of fullerene graphs with precisely 12 pentagonal faces. Moreover, the midpoints of its pentagonal faces form the planning of an icosahedron. They can be described by a vector  $(i, j)$ , determining the graph  $G_{i,j}$ . In 2013, Andova and Skrekovski presented and proved formulas to compute the diameter of the graphs  $G_{0,j}$  and  $G_{j,j}$ . Moreover, they presented a conjecture stating a lower bound for for the diameter of all fullerene graphs. Therefore, in this study, we investigate properties of fullerene graphs with icosahedral symmetry. We show that every graph  $G_{i,j}$  contains a reduced  $G_{0,j-i}$  and that every graph  $G_{i,j}$  is contained in an augmented  $G_{j,j}$ .

## A relationship between $D$ -eigenvalues and diameter

Renata R. Del-Vecchio, Miriam Abdón and Tayná Lobo

Fluminense Federal University, Brazil

**Keywords:** *distance spectrum; distance matrix; distance-regular graph*

In this poster we provide examples of connected graphs (which are not distance-regular) having diameter  $d$  and such that its distance matrix has less than  $d + 1$  eigenvalues ( $D$ -eigenvalues). This answers a question posed by Atik and Panigrahi in *On the distance spectrum of distance regular graphs, Linear Algebra and its Applications*, v. 478, p. 256–273, 2015.

## Conflict Free Closed Neighborhood Coloring Game

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**Keywords:** *conflict free coloring; coloring game; complete graphs; closed neighborhoods*

In Cellular Networks, communication between bases and mobile devices is established via radio frequencies. Interference occurs if one particular device communicates with two different bases that have the same frequency. So, every device must contact a base with an unique frequency and, since having a lot of different frequencies is expensive, it's important trying to minimize their quantity, in a way that there exists no interference. With that motivation, in 2002, Even, Lotker, Ron and Smorodinsky introduced the concept of Conflict Free coloring in a geometric scenario, which itself led to the study of Conflict Free Closed Neighborhood (CFCN) coloring in graphs. Inspired by this problem and by the well known coloring game, we introduce a game theoretical approach to CFCN coloring, and determine the minimum number of colors necessary for Alice to have a winning strategy in the case of Complete Graphs.

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## Teaching Newton's Binomial with Genetics

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**Keywords:** *combinatorics; Newton's binomial; mathematics education*

The teaching of Combinatorial Analysis is still done in a very mechanical way by some teachers who memorize formulas without a real content domain. This practice is repeated superficially and does not stimulate combinatorial reasoning. In this work, we show how Newton's Binomial is presented in Genetics, as a contribution to the teaching of the algebraic expansion of powers of a binomial.

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## Strong pseudoachromatic number of split graphs

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Federal University of Technology – Paraná, Brazil

**Keywords:** *strong pseudoachromatic coloring; strong pseudoachromatic number; non-proper complete coloring; split graphs*

A vertex coloring of a graph  $G$  is a strong pseudoachromatic coloring if there are adjacent vertices colored  $i$  and  $j$  for every two colors  $i$  and  $j$ , distinct or not. The maximum number of colors for a strong pseudoachromatic coloring of a graph  $G$  is the strong pseudoachromatic number,  $\psi^*(G)$ . Liu, Li, and Liu (2015) present upper and lower bounds for  $\psi^*(G)$  and determine  $\psi^*(G)$  when  $G$  is a complete graph, a path, a cycle, a complete multipartite graph, a complete bipartite graph from which a perfect matching is deleted, a wheel, or a fan. We observe that  $\psi^*(G) = \alpha'(G)$  for complete graphs and multipartite graphs, where  $\alpha'(G)$  is the size of a maximum matching of  $G$ , and pose the question of which other graph classes have  $\psi^*(G) = \alpha'(G)$ . We prove that split graphs is one of these classes.

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## Enumeration of cospectral and coinvariant graphs

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**Keywords:** *cospectral graph; coinvariant graph; Smith normal form; distance Laplacian; distance signless Laplacian*

We present enumeration results on the number of connected graphs up to 10 vertices for which there is at least one other graph with the same spectrum (cospectral mate), or at least one other graph with the same Smith Normal Form (coinvariant mate) with respect to several matrices associated to a graph. The presented numerical data give some indication that possibly the Smith Normal Form of the distance Laplacian and the distance signless Laplacian matrices could be a finer invariant than the spectrum for distinguishing graphs.

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## Intersection models for 2-thin and proper 2-thin graphs

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**Keywords:** *boxicity 2; proper thinness; thinness; VPG graphs*

The (proper) thinness of a graph is a width parameter that generalizes (proper) interval graphs, which are exactly the graphs of (proper) thinness one, and graphs with thinness two include bipartite convex graphs. Many NP-complete problems can be solved in polynomial time given a thin representation of bounded size. It is known that the thinness of a graph is at most its pathwidth plus one, and we prove that the proper thinness of a graph with at least one edge is at most its bandwidth. Another known result is that boxicity is a lower bound for thinness. The main result of this work are characterizations of (proper) 2-thin graphs as intersection graphs of rectangles in the plane, with appropriate conditions. An alternative characterization leads to showing that 2-thin graphs are L-graphs (thus  $B_1$ -VPG), and 3-thin graphs are  $B_3$ -VPG. It is also shown that  $B_0$ -VPG graphs may have arbitrarily large thinness.

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## A Near-tight Bound for the Rainbow Connection Number of Snake Graphs

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**Keywords:** *rainbow connection number; rainbow coloring; rainbow connectivity; snake graph*

The rainbow connection number of a connected graph  $G$  is the least  $k$  for which  $G$  admits a (not necessarily proper)  $k$ -edge-coloring such that between any pair of vertices there is a path whose edge colors are all distinct. This parameter has important applications. We present a near-tight bound for the rainbow connection number of snake graphs, a class commonly studied in labeling problems.

## Reduced indifference graphs are type 1

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**Keywords:** *total chromatic number; total coloring; reduced indifference graphs*

A total coloring is an assignment of colors to the vertices and edges of a graph  $G$  such that no two adjacent or incident elements receive the same color. The minimum number of colors for a total coloring of  $G$  is the total chromatic number,  $\chi''(G)$ . In the year that Celina Figueiredo, João Meidanis and Célia de Mello celebrate another decade of life, we prove an immediate consequence of their papers: all reduced indifference graphs have  $\chi''(G) = \Delta(G) + 1$ .

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## On Embedding Trees in Grids

Vitor T. F. de Luca<sup>1</sup>, Fabiano S. Oliveira<sup>1</sup> and Jayme L. Szwarcfiter<sup>1,2</sup>

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**Keywords:** *grid embedding; number of bends; EPG representations*

We are interested in embedding trees  $T$  with maximum degree at most 4 in a rectangular grid, such that the vertices of  $T$  correspond to grid points, while edges of  $T$  correspond to non-intersecting straight segments of the grid lines. The aim is to minimize the maximum number of bends of a path of  $T$ . We provide a quadratic-time algorithm for this problem. By applying this algorithm, we obtain an upper bound on the number of bends of EPG representations of graphs that are in both the classes of VPT and EPT.

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## $B_1$ -EPG representations using block-cutpoint trees

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**Keywords:** *edge intersection graph; block-cutpoint trees; block graph; cactus graph*

In this paper, we are interested in the edge intersection graphs of paths of a grid where each path has at most one bend, called  $B_1$ -EPG graphs and first introduced by Golumbic et al (2009). We also consider a proper subclass of  $B_1$ -EPG, the L-EPG graphs, which allows paths only in “L” shape. We show that two superclasses of trees are  $B_1$ -EPG (one of them being the cactus graphs). On the other hand, we show that the block graphs are L-EPG and provide a linear time algorithm to produce L-EPG representations of generalization of trees. These proofs employed a new technique from previous results based on block-cutpoint trees of the respective graphs.

## Contact L-graphs and their relation with planarity and chordality

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**Keywords:** *vertex intersection graphs; contact graphs; planar Laman graphs; chordal graphs; forbidden induced subgraphs*

An L-graph is a vertex intersection graph of paths in a grid such that all the paths in the representation have the shapes  $\{|\_,\perp\}$ . We will say that the graph is a strict L-graph if the paths only have the shape  $\perp$ . An (strict) L-contact graph is an (strict) L-graph such that all the paths in the representation do not cross each other and do not share an edge of the grid. The class of planar Laman graphs is of interest due to the fact that it contains several large classes of planar graphs. It was shown that planar Laman graphs are strict L-contact graphs but using all four rotations of  $\perp$ . In this work we study relations between (strict) L-graphs and planarity. We also present a minimal forbidden induced subgraph characterization of strict L-contact graphs within the class of chordal graphs.

## Gráficas cuadrado-complementarias 4-regulares de cuello grande

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**Keywords:** *graph equations; square-complementary graphs; regular graphs*

A square-complementary graph is a graph such that its square and its complement are isomorphic. Here, we provide a computer-assisted proof showing that there are no 4-regular square graphs of girth greater than 4, thus solving in the negative an open problem in Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. 5 (1994) 43-48, and also in Discrete Mathematics 327 (2014) 62-75.

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## Total Coloring in Some Split-Comparability Graphs

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**Keywords:** *total coloring; edge coloring; split graphs; comparability graphs*

A total coloring for a graph  $G$  is an assignment of colors to the edges and vertices of  $G$ , such that any pair of adjacent or incident elements have different colors. The least number of colors for which  $G$  has a total coloring is denoted  $\chi''(G)$ . It is known that split-comparability graphs have  $\chi''(G) \leq \Delta(G) + 2$ . In this work we show that certain split-comparability graphs with odd maximum degree have  $\chi''(G) = \Delta(G) + 1$ .

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## $L(2, 1)$ -labeling of Loupekine Snarks

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**Keywords:**  $\lambda$ -labelings; Loupekine snarks; vertex coloring

An  $L(2, 1)$ -labeling of a graph  $G$  is a function  $f : V(G) \rightarrow [0, t]$  such that  $|f(u) - f(v)| \geq 2$  if  $d(u, v) = 1$ , and  $f(u) \neq f(v)$  if  $d(u, v) = 2$ , for  $u, v \in V(G)$ . The span of an  $L(2, 1)$ -labeling is the largest integer  $t$  used. An  $L(2, 1)$ -labeling of  $G$  is optimal when it has the smallest possible span and such a value is denoted by  $\lambda(G)$ . Georges and Mauro explored the problem of determining  $\lambda(G)$  for 3-regular graphs. The authors conjectured that, with the exception of the Petersen Graph, every connected 3-regular graph  $G$  has  $\lambda(G) \leq 7$ . In this work, we verify Georges and Mauro's Conjecture for Loupekine Snarks and we present a lower bound on  $\lambda(G)$  for the members of this family of graphs.

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## Two Level Hamming-Huffman Trees

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**Keywords:** *Hamming-Huffman tree; hypercube; minimum neighborhood; algorithm*

In this work, we describe an algorithm to determine optimal two level Hamming-Huffman trees when the symbols have uniform frequencies. That is, the algorithm builds optimal Hamming-Huffman trees in which all leaves lay in at most two different levels. Also, considering experimental results, we conjecture that, for uniform frequencies, optimal two levels Hamming-Huffman trees are optimal in general.

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## A result on total coloring of fullerene nanodiscs

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**Keywords:** *cubic graph; total coloring; Fullerene nanodisc*

A graph is a mathematical model used to represent relationships between objects. The general characters that both of these objects and its relationships can assume, allowed the construction of the so-called Graph Theory, which has been applied to model problems in several areas, such as Mathematics, Physics, Computer Science, Engineering, Chemistry, Psychology and industry. Most of them are large scale problems. Fullerene graphs are mathematical models for carbon-based molecules experimentally discovered in the early 1980s by Kroto, Heath, O'Brien, Curl and Smalley. Many parameters associated with these graphs have been discussed to describe the stability of Fullerene molecules. By definition, Fullerene graphs are cubic, planar, 3-connected with pentagonal and hexagonal faces. The motivation of the present study is to find an efficient method to obtain a 4-total coloring of a particular class of Fullerene graphs named Fullerene nanodiscs, if it exists.

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## Alliance and Domination on Uniformly Clique-expanded Graphs

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**Keywords:** *domination; alliance; clique-expanded graph*

This work aims at presenting the uniformly clique-expanded graphs and its results on global defensive alliance and total dominating set problems. Those graphs are related to Sierpiński graphs (KLAVŽAR et. al, 2002) and subdivided-line graphs (HASUNUMA, 2013). We show the minimum cardinality of the global defensive alliance for some particular situations of uniformly clique-expanded graphs, and we also relate that cardinality to the total dominating set number for graphs having a path or cycle as the root.

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## Sharp Bounds for the Annihilation Number of the Nordhaus-Gaddum type

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**Keywords:** *annihilation number; Nordhaus-Gaddum problem; interval property; extremal problems*

The annihilation number is a graph invariant used as a sharp upper bound for the independence number. In this work, we present bounds and Nordhaus-Gaddum type inequalities for the annihilation number. We also investigate the extremal behavior of the invariant and showed that both parameters satisfy the interval property. In addition, we characterize some extremal graphs, ensuring that the bounds obtained are the best possible.

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## Un algoritmo lineal para el problema de la $k$ -upla dominación en grafos web

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**Keywords:** *problemas de dominación; algoritmo eficiente; grafos web*

Un vértice de un grafo se domina a sí mismo y a todos sus vértices adyacentes. Para un grafo  $G = (V; E)$  y un número entero positivo  $k$ , un conjunto  $k$ -upla dominante en  $G$  es un subconjunto  $D \subseteq V$  tal que todo vértice en  $V$  es dominado por al menos  $k$  elementos de  $D$ . El problema de la  $k$ -upla dominación consiste en hallar un conjunto  $k$ -upla dominante en  $G$  de mínimo tamaño ( $\gamma_k(G)$ ). Este problema es NP-difícil. Estudiamos subclases de grafos arco-circulares, en los cuales la complejidad de este problema no es conocida para  $k \neq 1$ . En un trabajo previo exploramos la subclase de los grafos concave-round, habiendo comenzado nuestro estudio sobre los grafos web. Obtuvimos  $\gamma_k(W)$  para todo grafo web  $W$ . En este poster mostramos el diseño de un algoritmo lineal que devuelve un conjunto  $k$ -upla dominante de tamaño  $\gamma_k(W)$  para todo  $k$  y grafo web.

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## Rotulação $\mathcal{L}(h, k)$ dos Sunlets

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**Keywords:** *teoria estrutural de grafos; rotulação de grafos; rotulação  $\mathcal{L}(h, k)$ ; rotulação  $\mathcal{L}(2, 1)$*

Uma rotulação  $\mathcal{L}(h, k)$  de um grafo simples  $G$  é uma atribuição de inteiros não-negativos aos vértices de  $G$  tal que a diferença entre os rótulos de vértices adjacentes é pelo menos  $h$ , e de vértices que tenham um vizinho em comum é pelo menos  $k$ . O span de  $G$  é o menor inteiro  $t$  para o qual  $G$  admite uma rotulação  $\mathcal{L}(h, k)$  em que a diferença entre os rótulos de quaisquer dois vértices é no máximo  $t$ . Neste trabalho, determinamos o span dos Sunlets, que são os grafos obtidos a partir dos ciclos adicionando-se um pingente a cada vértice.

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## Emparelhamentos perfeitos no produto cartesiano de árvores

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**Keywords:** *produto cartesiano de grafos; emparelhamento; árvore*

Neste trabalho, investiga-se a existência de emparelhamento perfeito no produto cartesiano de duas árvores sem emparelhamento perfeito, com foco no caso de árvores do tipo caterpillar. Especificamente, é descrita uma família infinita de caterpillars com um número par de vértices e sem emparelhamento perfeito, tais que o produto cartesiano de duas quaisquer destas árvores possui emparelhamento perfeito.

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## Connected Matchings

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**Keywords:** *graph; matching; connected*

Graph matching problems are well known and studied, in which we want to find sets of pairwise non-adjacent edges. This work focus on the study of matchings that induce subgraphs with special properties. For this work, we consider the property of being connected, also studying it for weighted or unweighted graphs. For the unweighted ones, we want to obtain a matching with the maximum cardinality, while for the weighted graphs, we look for a matching whose sum of the matching edge weights is maximum. The problem of maximum connected matching is polynomial. We show ideas that lead to two linear algorithms. One of them, having a maximum matching as input, determines an maximum unweighted connected matching. The complexity of the maximum weighted connected matching problem is unknown for general graphs. However, we present a linear time algorithm that solves it for trees.

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## Number of spanning trees of a subclass of matrogenic graphs

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**Keywords:** *spanning trees; matrogenic graphs; Laplacian spectrum*

In this work, we study a subclass of the family of matrogenic graphs. This family is little known in the literature, and does not yet have its determined Laplacian spectrum. With the intention of beginning the study on the Laplacian spectrum of this family, we will show the spectrum of a subclass contained therein, and then using the Matrix-Tree Theorem, its number of spanning trees.

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## Arc-disjoint Branching Flows: a study of necessary and sufficient conditions

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**Keywords:** *flows; arc-disjoint flows; branching-flows; digraphs*

An  $s$ -branching flow  $f$  in a network  $N = (D, c)$  (where  $c$  is the capacity function) is a flow that reaches every vertex in  $V(D) - s$  from  $s$  while losing exactly one unit of flow in each vertex other than  $s$ . In other words, for a vertex  $v \in V(D) - s$ , the difference between the flow entering and leaving  $v$  is one. We further investigate a conjecture proposed by Costa et. al (2019) that aims to characterize networks admitting  $k$  arc-disjoint  $s$ -branching flows, generalizing a result that provides such characterization when all arcs have capacity  $n - 1$ , based on Edmonds' branching Theorem. We show that, in general, the conjecture is false. However, it holds for out-branchings with parallel arcs.

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## Maximal Independent Sets in Graphs of Girth at Least 6

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**Keywords:** *trees; independent sets; independence gap; well-covered graphs*

The independence gap of a graph  $G$  is the difference between the maximum and the minimum sizes of a maximal independent set in  $G$ . We present a characterization of graphs with independence gap at least 1 that are of girth at least 6 with exactly two support vertices having exactly  $r$  leaves. We also present results related to the number of trees with specific sizes of maximal independent sets.

## The 3-flow conjecture for almost even graphs with up to six odd vertices

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**Keywords:** *integer flows; modular flows; 3-flow Conjecture*

In this work, our objective is to characterize classes of graphs with up to four 3-cuts that admit a 3-flow.  $K_4$ , the complete graph on four vertices, is the smallest bridgeless graph that does not admit a 3-flow. We focus on almost even graphs, i.e., bridgeless graphs with up to six odd vertices. We obtain a characterization for almost even graphs with up to four odd vertices. We also obtain a partial characterization for almost even graphs with up to four 3-vertices (vertices of degree three) and two odd vertices of higher degree.

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## Lexicographic Breadth-First Search and Exact Algorithms for the Maximum Clique Problem

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**Keywords:** *lexicographic breadth-first search; maximum clique problem; algorithms; experimental analysis of algorithms*

There are algorithms that use the branch and bound technique for Maximum Clique Problem (CM) and Vertex Coloring as the upper bound. The algorithm known as Lexicographic Breadth-first Search (LexBFS) is capable of producing vertex ordering of some graph classes that lead to the solution of difficult problems in polynomial time. The aim of this work is to study branch and bound algorithms that use vertex coloring to solve CM modified with the LexBFS. Methods of Experimental Analysis of Algorithms were used to evaluate changes. It was possible to make inferences about the results obtained using the statistical method of hypothesis testing. The conclusion is that in instances of graphs considered the search space is smaller, but the execution time was longer. When evaluating with generated random instances, both the search space and the execution time were significantly shorter for some algorithms.

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## 2-Independent Sets in Complementary Prisms

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**Keywords:** *k-independent; k-dominating; complementary prism*

The subset  $S$  is  $k$ -independent if the maximum degree of the subgraph induced by the vertices of  $S$  is at most  $k - 1$ . We present sharp lower and upper bounds for maximum 2-independent sets in complementary prism of any graph, characterize the graphs for which the upper and lower bound holds, and present closed formulas for the complementary prism of paths, cycles and complete graphs.

## Kernelization lower bounds for Multicolored Independent Set

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**Keywords:** *multicolored independent set; kernelization; parameterized complexity; cross-composition*

In the Multicolored Independent Set problem, we are given a graph  $G$  and a partition of  $V(G)$  into  $k$  sets, and the goal is to find an independent set of  $G$  that hits each part exactly once. Along with Independent Set, Clique, and Multicolored Clique, Multicolored Independent Set is one of the most widely used problems in lower bound proofs in parameterized complexity. While all four are known to be fixed-parameter tractable when parameterized by vertex cover, only the latter had no kernelization results. We fill this gap by using the cross-composition framework of Bodlaender et al. [STACS 2011] to show that Multicolored Independent Set has no polynomial kernel when parameterized by distance to any non-trivial graph class and size of the solution, unless NP is contained in coNP/poly

## Partitioning Graphs into Monochromatic Trees

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**Keywords:** *computational complexity; monochromatic trees; vertex disjoint*

The Partitioning Graphs into Monochromatic Trees problem (PGMT) consists of partitioning an edge-colored graph into vertex disjoint monochromatic trees. It has been known that PGMT is NP-Complete. In this work we consider some particular cases of this problem, where the number of occurrences of each color is bounded by a constant  $f$ . We show that this new version is NP-Complete even for  $f = 3$ .

## $A_\alpha$ -Spectrum of some Matrogenic Graphs

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**Keywords:**  $A_\alpha$ -characteristic polynomial; eigenvalues; matrogenic graphs

Let  $G$  be a connected graph of order  $n$ ,  $A(G)$  the adjacency matrix of  $G$  and  $D(G)$  the diagonal matrix of the row-sums of  $A(G)$ . For every real  $\alpha \in [0, 1]$ , Nikiforov defined the matrix  $A_\alpha(G)$  as  $A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G)$ . In this work, we show a factorization of the  $A_\alpha$ -characteristic polynomial of a family of matrogenic graphs.

## Decomposing cubic graphs into locally irregular subgraphs

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**Keywords:** decomposition; locally irregular coloring; graph theory; cubic graph

A locally irregular graph is a graph in which adjacent vertices have distinct degrees. A locally irregular  $k$ -coloring, or  $k$ -LIC, for short, of a graph  $G$  is a coloring of  $E(G)$  in which every color class induces a locally irregular subgraph. Baudon, Bensmail, Przybyło, and Woźniak, (2015) conjectured that if a graph  $G$  admits a  $k$ -LIC, then  $G$  admits a 3-LIC. In this work we verify this conjecture for a family of cubic graphs, and we present a condition for graphs not to admit a 2-LIC.

## Some results on Vertex Separator Reconfiguration

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**Keywords:** vertex separators; reconfiguration; PSPACE-hard; NP-hard

We present the first results on the complexity of the reconfiguration of vertex separators under the three most popular rules: token addition/removal, token jumping, and token sliding. We show that, aside from some trivially negative instances, the first two rules are equivalent to each other and that, even if only on a subclass of bipartite graphs, token sliding is not equivalent to the other two unless  $NP = PSPACE$ ; we do this by showing a relationship between separators and independent sets in this subclass of bipartite graphs, which implies that Vertex Separator is NP-hard under token jumping and PSPACE-hard under token sliding.

## Generalized packing functions of graphs with few $P_4$ 's

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**Keywords:** *generalized packing function; spider graphs;  $P_4$ -tidy graphs*

We introduce a concept of packing of graphs which generalizes all those previously defined in the literature and we study the computational complexity of computing the associated parameter, the generalized packing number of the graph. We find that this new packing parameter comes to be much more complicated to handle than those previously defined, even on particular graph classes as spider and quasi-spider graphs. Nevertheless, we prove that the associated optimization problem can be solved in linear time for some graph classes with few  $P_4$ 's.

## O número de dominação total e independência aberta para produto lexicográfico de grafos

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**Keywords:** *conjunto dominante total; conjunto independente aberto; produto lexicográfico*

Um conjunto dominante total de um grafo  $G = (V, E)$ , é um subconjunto  $D \subseteq V(G)$  tal que  $|N(v) \cap D| \geq 1$ , para todo  $v \in V(G)$ . E um conjunto independente aberto de um grafo  $G$  é um conjunto  $S$  de vértices de  $G$ , tal que  $|N(v) \cap S| \leq 1$ , para cada  $v \in S$ . Neste trabalho apresentamos resultados sobre o número de dominação total e independência aberta para o produto lexicográfico de grafos gerais e para classes simples como os ciclos  $C_n$ , caminhos  $P_n$  e completos  $K_n$ .

## Mutually Included Biclique Graphs of Interval Containment Bigraphs and Interval Bigraphs

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**Keywords:** *biclique; biclique graph; interval bigraph; interval containment bigraph*

The recognition of biclique graphs in general is still open. Recently, Groshaus and Guedes introduced the mutually included biclique graph as an intermediate operator to define the biclique graph. Also, we previously studied the biclique graph of interval bigraphs and proper interval bigraphs. In this work, we extend the results to a superclass, the interval containment bigraphs, in the context of the mutually included biclique graphs.

---

## Swapping Tokens on Graphs

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**Keywords:** *graph theory; reconfiguration; integer linear programming; cograph*

Reconfiguration problems focus on transforming a state of a combinatorial or geometric object into another by performing some sequence of operations. In one important class of reconfiguration problems, valid moves consist on moving items along the edges of a graph to achieve a desired final configuration. The Token Swapping (TS) problem is one of these problems and is known to be NP-hard. This work introduces the reader to the approach of solving this problem by an integer linear program model and present an efficient algorithm for solving the TS problem on complement-reducible graphs, also called cographs.

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## On the edge- $P_3$ and edge- $P_3^*$ -convexity of grid graphs

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**Keywords:** *Helly number; convexity;  $P_3$ -convexity;  $P_3^*$ -convexity*

Consider a graph  $G = (V, E)$  and a subset  $C \subseteq V$ . The  $P_3$ -convex hull (resp.  $P_3^*$ -convex hull) of  $C$  is obtained by iteratively adding vertices with at least two neighbors (two non-adjacent neighbors) in  $C$ . A subset  $S \subseteq V$  is  $P_3$ -Helly-independent ( $P_3^*$ -Helly-independent) when the intersection of the  $P_3$ -convex hulls ( $P_3^*$ -convex hulls) of  $S \setminus \{v\}$  ( $\forall v \in S$ ) is empty. The  $P_3$ -Helly number ( $P_3^*$ -Helly number) is the size of a maximum  $P_3$ -Helly-independent ( $P_3^*$ -Helly-independent). The edge counterparts of  $P_3$ -Helly-independent and  $P_3^*$ -Helly-independent of a graph follow the same restrictions applied to its edges. In this work, we established the edge  $P_3^*$ -Helly number of grid graphs  $G(p \times q)$  when both  $p$  and  $q$  are larger than 15. Moreover, we give partial results on forbidden configurations of edge  $P_3$ -Helly independent sets for these graphs.

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## Recognition of Biclique Graphs: What we know so far

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**Keywords:** *bicliques; biclique graphs; recognition problem; functors; graph classes*

The recognition of biclique graphs in general is still open. In recent years we presented some result on the characterization of biclique graphs of graphs of certain graph classes, along with the complexity associated to the recognition problem. Those results introduced some intermediate operators, which we call now as "functors". In this work we summarize all those results and organize the different approaches using the functors.

## Subclass Hierarchy on Circular Arc Bigraphs

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**Keywords:** *circular arc bigraphs; subclasses; Helly circular arc bigraphs; circular convex bipartite; doubly circular convex bipartite; non-bichordal Helly circular arc bigraphs; proper circular arc bigraphs; proper interval bigraphs*

A bipartite graph is a circular arc bigraph if there exists a bijection between its vertices and a family of arcs on a circle such that vertices of opposing partite sets are neighbors precisely if their corresponding arcs intersect. The class is a relatively unexplored subject, with most results on it and its subclasses being quite recent. In our work, we provide a full exploration of the containment relations and intersections between seven subclasses of circular arc bigraphs.

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## Efficient characterizations and algorithms of tree $t$ -spanners

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**Keywords:** *3-admissibility; stretch index; graphs with few  $P_4$ 's*

The  $t$ -admissibility problem has been widely studied specially because determining if a graph  $G$  is 3-admissible is still an open problem since it was proposed. Although, recognizing if a graph is 2-admissible is a polynomial-time solvable problem, we realized that for some classes could be easier, so in this work we present simple and efficient algorithms in order to characterize 2 and 3-admissible graphs for some graph classes as cographs, split graphs,  $P_4$ -sparse and other superclasses.

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## Hamiltonicity of Token Graphs of Some Fan Graphs

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**Keywords:** *double vertex graphs; Johnson graphs; Hamiltonian graphs*

In this poster we present some recent results about the Hamiltonicity of the 2-token graph,  $F_2(G)$ , and the 2-multiset graph,  $M_k(G)$  of some fan graphs. As a consequence, we exhibit an infinite family of graphs for which  $F_2(G)$  and  $M_2(G)$  are Hamiltonian.

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