

A Survey on Independent Sets in Graphs

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An *independent set* of $G = (V, E)$ is a subset S of V such that no two vertices are adjacent. $S \subseteq V$ is a *maximal independent set* (mis for short) if it is not properly contained in any other independent set of G . The family of mis of G is denoted by $M(G)$ and its cardinality is $\mu(G)$. Proding and Tichy [15] introduced the *Fibonacci number* $f(G)$ of a graph G as the number of independent sets of G , not necessarily maximal, including the empty set.

Valiant [18] showed that the problem of counting the number of mis is $\#\mathcal{P}$ -complete for a general graph. Füredi [3] determined a bound for $\mu(G)$ when G is a connected graph with at least 50 vertices. Independently, Griggs et al. [5] established the same bound and characterized the extremal graphs that attain this bound for graphs with six or more vertices. This problem has been studied for various graph classes, including trees (Wilf [19]), graphs with at most one cycle (Jou and Chang [8]), graphs with r cycles (Sagan and Vatter [16] and Goh et al. [4]), bipartite graphs (Liu [11]), triangle-free graphs (Hujter and Tuza [7] and Chang and Jou [1]) and unicyclic graphs (Koh et al. [9] and Pedersen and Vestergaard [14]). Euler [2] calculated $\mu(G)$ when G is a grid graph with up to five rows.

The problem of enumerating all maximal independent sets of a graph has also been considered by different authors. For a general connected graphs Tsukiyama et al. [17] developed the best algorithm. Leung [10] enumerated all mis of interval graphs, chordal graphs and circular arc graphs. Yu and Chen [20] solved the problem for permutation graphs. Hota et al. [6] did the same for trapezoid graphs.

The intersection graph $\mathcal{I}(G)$ of the family of all maximal independent sets of a graph G is called the *Independent Graph* of G . Each vertex of $\mathcal{I}(G)$ corresponds to a mis of G and two vertices are adjacent in $\mathcal{I}(G)$ if their corresponding mis have non-empty intersection in G . Analogously, the *Clique Graph* $\mathcal{K}(G)$ is the intersection graph of all cliques of G . Ortiz and Villanueva determined $\mu(G)$ and generated the family of mis in a caterpillar graph and characterized the independent graph [12]. They did the same for grid graphs with two rows [13].

In this work we survey results on these problems related to independent sets: counting independent sets and enumerating maximal independent sets. .

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