A relationship between D-eigenvalues and diameter.

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Objective

Our goal is to provide examples of connected graphs having diameter $d$ and less than $d+1$ D-eigenvalues. This answers a question stated by Atik and Panigrahi in [4, Problem 4.3].

Introduction

It is known, by [1], that if $G$ is a graph of diameter $d$ then the adjacency matrix of $G$ has at least $d+1$ distinct eigenvalues. We can see in [2] that distance-regular graphs actually attains this minimum, that is, they have exactly $d+1$ distinct adjacency eigenvalues.

A simple connected graph $G$ is called distance-regular if it is regular, and if for any two vertices $x, y \in V(G)$ at distance $i$, there are constant numbers of neighbors $c_i$ and $b_i$ of $y$ at distance $i−1$ and $i+1$ from $x$, respectively.

Figure 1: $C_5$ and Petersen graph are examples of distance regular graphs. More generally, $C_n$ is a distance regular graph.

It seems reasonable to ask whether these results can be extended to the eigenvalues associated with the distance matrix $(D$-eigenvalues) of a simple connected graph $G$. Indeed, Lin et al. [5] ask if, for a graph $G$ with diameter $d$, its distance matrix has at least $d+1$ distinct eigenvalues. Atik and Panigrahi give a negative answer to this problem in [4]. Moreover, they prove that a distance-regular graph with diameter $d$ has at most $d+1$ distinct $D$-eigenvalues and leave the following question: “Are there connected graphs other than distance regular graphs with diameter $d$ and having less than $d+1$ distinct $D$-eigenvalues?”

In what follows, we answer this question positively by giving two examples of connected graphs with diameter $d$ having less than $d+1$ distinct $D$-eigenvalues.

Examples

In our example we consider two bipartite graphs $G_1$ and $G_2$ described in figures 2 and 3. We have that $|V(G_1)| = 20$ and $|V(G_2)| = 70$, and that $diam(G_1) = 5$ and $diam(G_2) = 7$. However, both graphs have exactly four distinct $D$-eigenvalues. These graphs and their respectively $D$-spectrum are shown as follows.

In particular, our examples are both distance-biregular graphs, for a precise definition see [3].

\[ spect(G_1) = \begin{bmatrix} 50 & 0 & -2 & -12 \\ 1 & 14 & 1 & 4 \end{bmatrix} \]

\[ spect(G_2) = \begin{bmatrix} 245 & 0 & -5 & -40 \\ 1 & 62 & 1 & 6 \end{bmatrix} \]

Conclusions

About the problem proposed by Atik and Panigrahi in [4], it can be said that there are other connected graphs with diameter $d$, in addition to distance regular graphs, having less than $d+1$ distinct $D$-eigenvalues. More specifically, the graphs presented in this work have exactly 4 distinct $D$-eigenvalues. For future works, we are interested in characterize a class of distance-biregular graphs with this property.

References


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