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A relationship between D-eigenvalues and diameter.

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Objective

Our goal is to provide examples of connected graphs having diameter d and less than d + 1 D-eigenvalues. This answers a question stated by Atik and Panigrahi in [4, Problem 4.3].

Introduction

It is known, by [1], that if G is a graph of diameter d then the adjacency matrix of G has at least $d+1$ distinct eigenvalues. We can see in [2] that distance-regular graphs actually attains this minimum, that is, they have exactly $d+1$ distinct adjacency eigenvalues.

A simple connected graph G is called *distance-regular* if it is regular, and if for any two vertices $x, y \in V(G)$ at distance i, there are constant number of neighbors c_i and b_i of y at distance $i-1$ and $i+1$ from x, respectively.

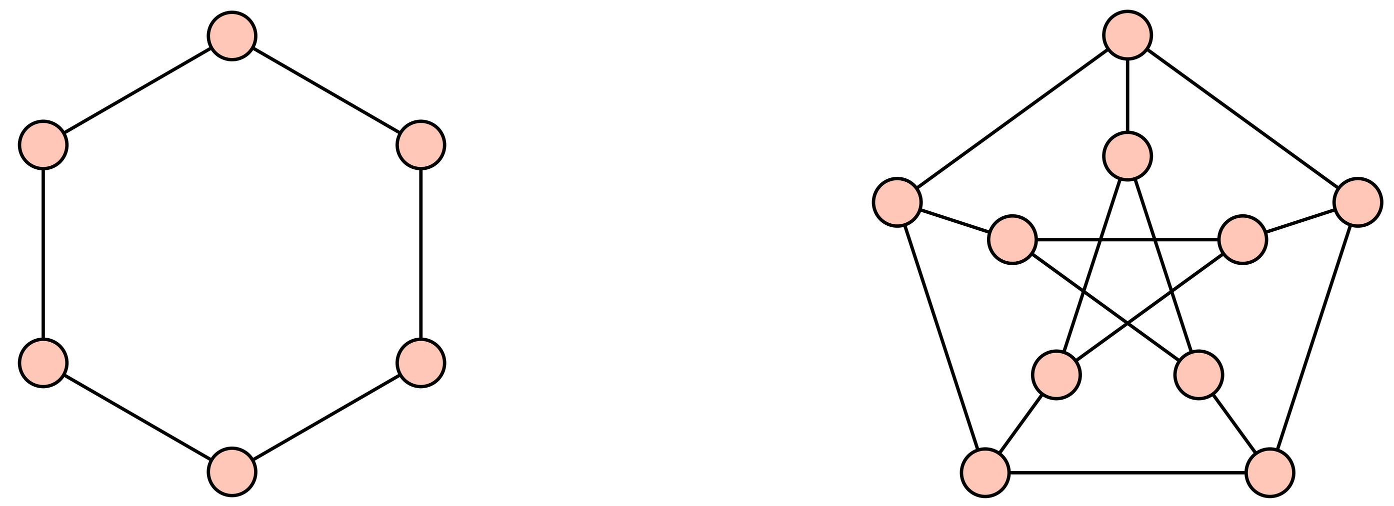

Figure 1: C_6 and Petersen graph are examples of distance regular graphs. More generally, C_n is a distance regular graph.

It seems reasonable to ask whether these results can be extended to the eigenvalues associated with the distance matrix (D-eigenvalues) of a simple connected graph G. Indeed, Lin et al. [5] ask if, for a graph G with diameter d, its distance matrix has at least $d+1$ distinct eigenvalues. Atik and Panigrahi give a negative answer to this problem in [4]. Moreover, they prove that a distance-regular graph with diameter d has at most $d+1$ distinct D-eigenvalues and leave the following question: "*Are there connected graphs other than distance regular graphs with diameter d and having less than $d+1$ distinct D-eigenvalues?*".

In what follows, we answer this question positively by given two examples of connected graphs with diameter d having less than $d+1$ distinct D-eigenvalues.

Examples

In our example we consider two bipartite graphs G_1 and G_2 described in figures 2 and 3. We have that $|V(G_1)| = 20$ and $|V(G_2)| = 70$, and that $diam(G_1) = 5$ and $diam(G_2) = 7$. However, both graphs have exactly four distinct D-eigenvalues. These graphs and their respectively D-spectrum are shown as follows.

In particular, our examples are both *distance-biregular* graphs, for a precise definition see [3].

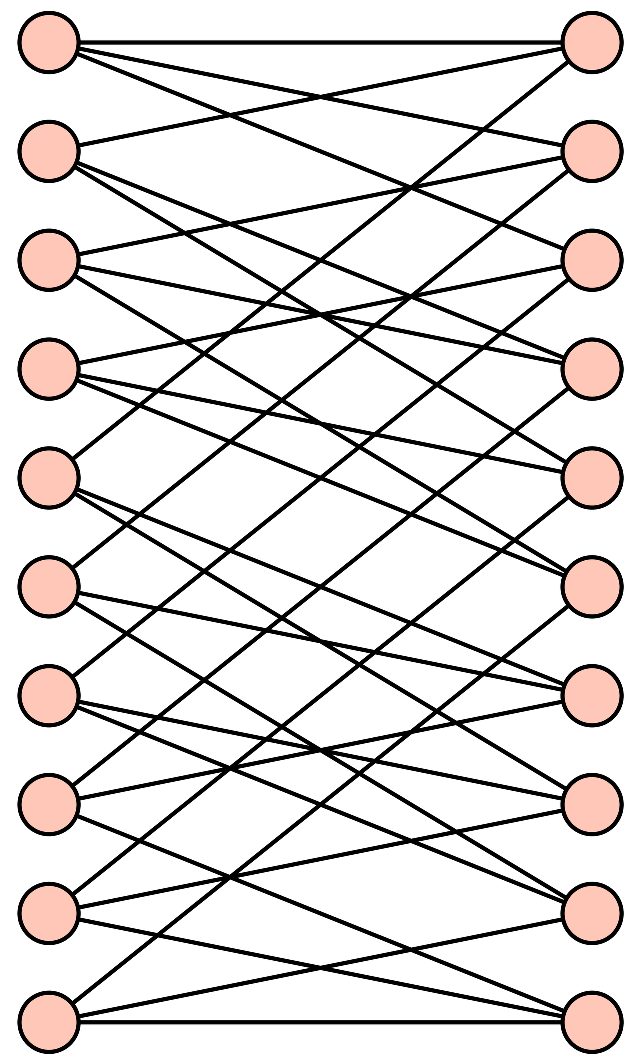

Figure 2: The graph G_1.

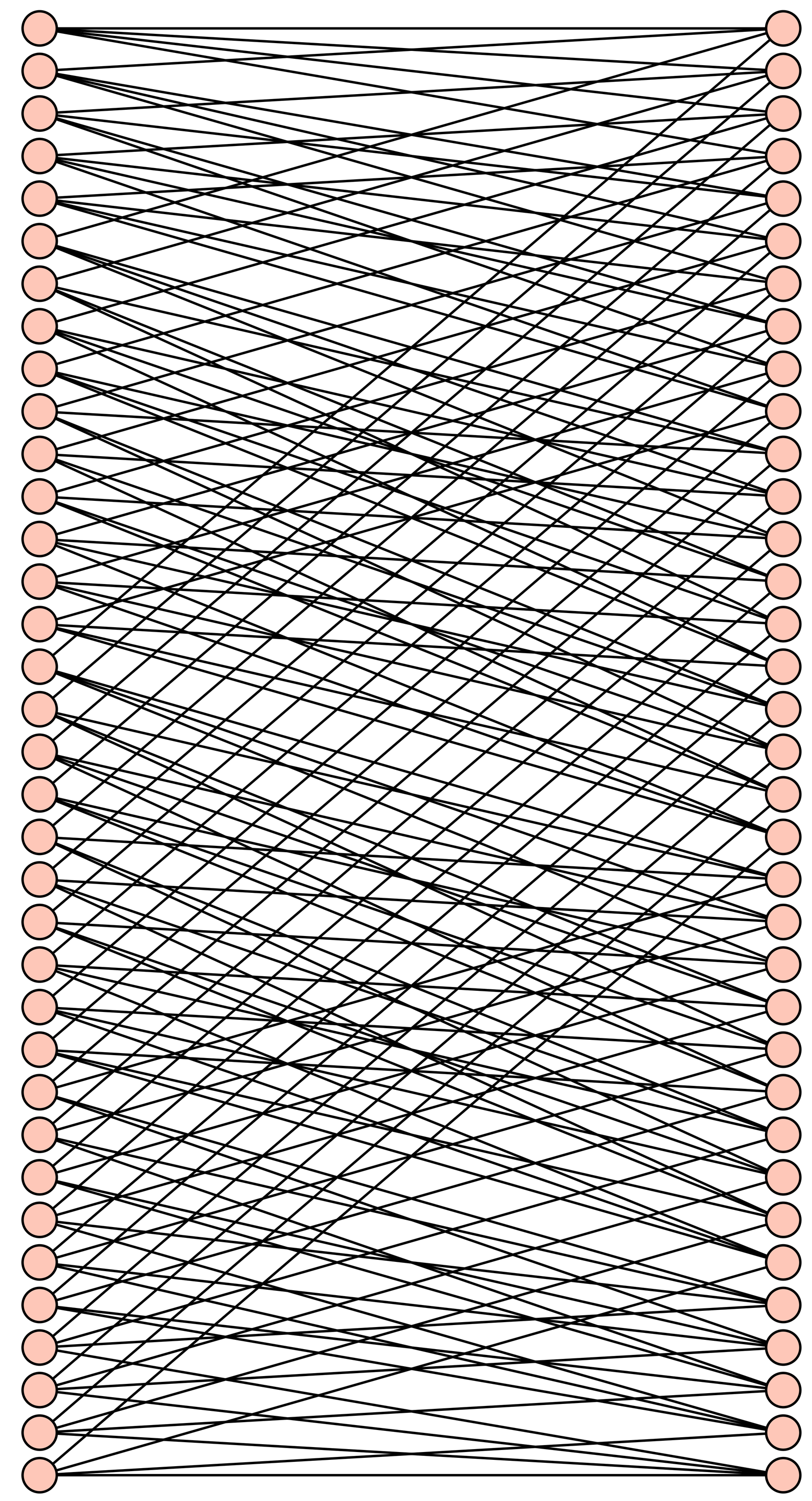

Figure 3: The graph G_2.

$$spect\,(G_1) = \begin{bmatrix} 50 & 0 & -2 & -12 \\ 1 & 14 & 1 & 4 \end{bmatrix}$$

$$spect\,(G_2) = \begin{bmatrix} 245 & 0 & -5 & -40 \\ 1 & 62 & 1 & 6 \end{bmatrix}$$

Conclusions

About the problem proposed by Atik and Panigrahi in [4], it can be said that there are other connected graphs with diameter d, in addition to distance regular graphs, having less than $d+1$ distinct D-eigenvalues. More specifically, the graphs presented in this work have exactly 4 distinct D-eigenvalues. For future works, we are interested in characterize a class of distance-biregular graphs with this property.

References

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[3] C. Delorme, Distance biregular bipartite graphs, European jornal of Combinatorics, v. 15, p. 223–238, 1994.

[4] F. Atik, P. Panigrahi, On the distance spectrum of distance regular graphs, Linear Algebra Appl., v. 478, p. 256–273, 2015.

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Acknowledgment