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Paths On Hosts: B_1-EPG, EPT and VPT Graphs

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Introduction

This research contains as a main result the proof that every Chordal B1-EPG graph is simultaneously in the VPT and EPT graph classes. In addition, we describe a set of graphs that defines Helly-B_1-EPG families. In particular, this work presents some features of non-trivial families of graphs properly contained in Helly-B_1 EPG, namely Bipartite, Block, Cactus and Line of Bipartite graphs.

Objective

In this work we will mainly explore the EPG graphs, in particular B_1-EPG graphs. However, other classes of intersection graphs will be studied such as EPT and VPT graph classes.

Definitions and Technical Results

- A graph is a B_k-EPG graph if it admits an EPG representation in which each path has at most k bends;
- When k = 1 we say that this is a **single bend EPG representation** or simply a B_1-EPG representation;
- In a B_1-EPG representation, a clique K can be **edge-clique** or **claw-clique** [3].

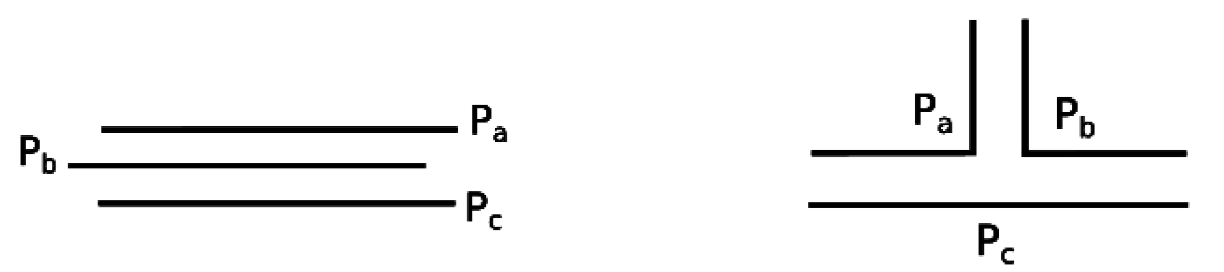

Figure 1: Representation of a clique as edge-clique and as claw-clique.

- A collection of sets satisfies the **Helly property** when every pairwise intersecting sub-collection has at least one common element;
- When this property is satisfied by the set of paths used in a representation, we get a Helly representation;
- Helly-B_1-EPG graphs were studied in [2];
- EPG, EPT and VPT representations arise in circuit layout problems and layout opt imization [4];
- VPT and EPT graphs are the vertex-intersection and edge-intersection graphs of paths on trees, respectively;
- VPT and EPT graphs are incomparable families of graphs.

Subclasses of Helly-B1-EPG Graphs

Theorem 1: Let G be a B_1-EPG graph. If G is $\{S_3, S_{3'}, S_{3''}, C_4\}$-free then G is a Helly-$B_1$-EPG graph.

(a) Claw with paths. **(b) Subgraph induced by paths.**

Figure 2: Reconstruction of intersection model.

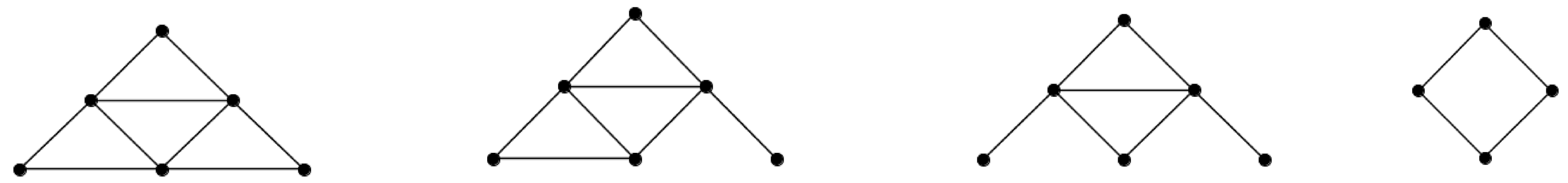

Figure 3: Graphs on statement of Theorem 1: S_3, $S_{3'}$, $S_{3''}$, C_4.

- Bull-free graphs are $\{S_3, S_{3'}, S_{3''}\}$-free, so these results implies in results of [1].

Theorem 2: If the graph G is B_1-EPG and diamond-free then G is Helly-B_1-EPG.
Corollary: Bipartite, Block, Cactus and Line of Bipartite graphs are Helly-B1-EPG.

Relationship among Chordal B1-EPG, VPT and EPT graphs

Theorem 3: Chordal B_1-EPG $\subsetneq$ VPT. *Theorem 4: Chordal B_1-EPG $\subsetneq$ EPT.*

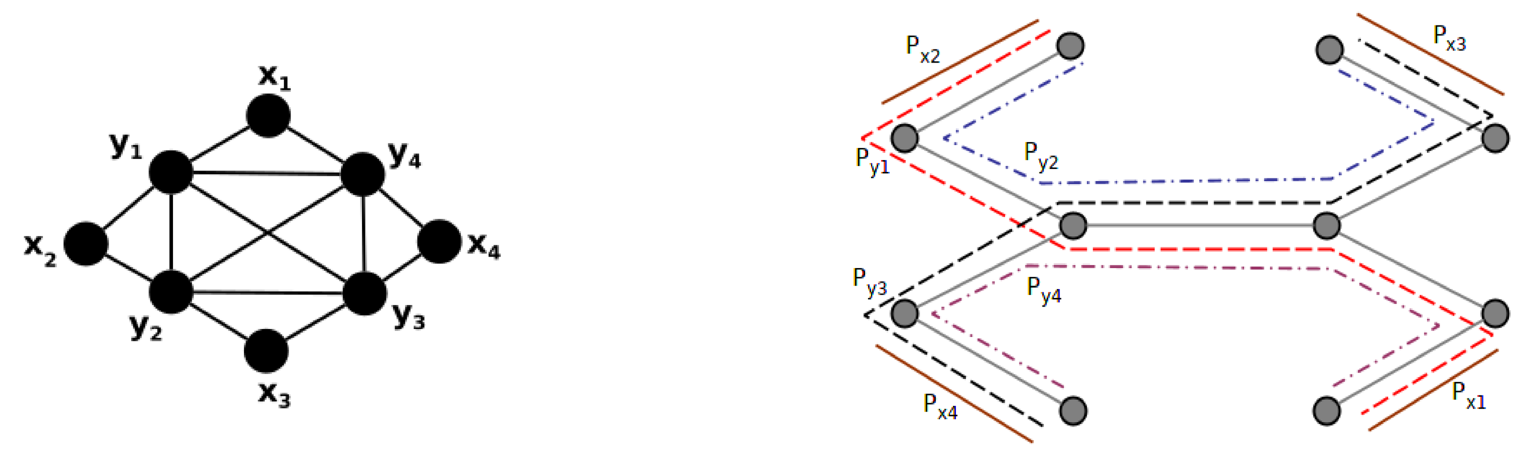

Figure 4: Graph S_4 and one of its possible VPT and EPT representations.

References

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[2] C.F. Bornstein, M.C. Golumbic, T.D. Santos, U.S. Souza, and J.L. Szwarcfiter. The Complexity of Helly-B_1-EPG Graph Recognition. Discrete Mathematics & Theoretical Computer Science, 22 (2020), https://dmtcs.episciences.org/6506/pdf.
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