A Near-tight Bound for the Rainbow Connection Number of Snake Graphs★

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Introduction

The *rainbow connection number* of a connected graph G, denoted $rc(G)$, is the least k for which G admits a (not necessarily proper) k-edge-coloring such that between any pair of vertices there is a path whose edge colors are all distinct. This parameter has important applications [3].

Remark ([1]) *If G is a connected and not trivial graph with n vertices, then $diam(G) \leq rc(G) \leq |E(G)|$.*

We present a near-tight bound for the rainbow connection number of snake graphs, a class commonly studied in labeling problems [2, 5].

Let $\ell \geq 3, k \geq 1, n \geq 2$. An *$\ell$-gon k-multiple snake graph* over n vertices, denoted $S(\ell, k, n)$, is obtained from $P_n\colon v_0 v_1 \ldots v_{n-1}$ by adding k multiple edges between v_i and v_{i+1} for $0 \leq i \leq n-2$ and making $\ell - 2$ successive subdivisions at each edge added. See Fig. 1.

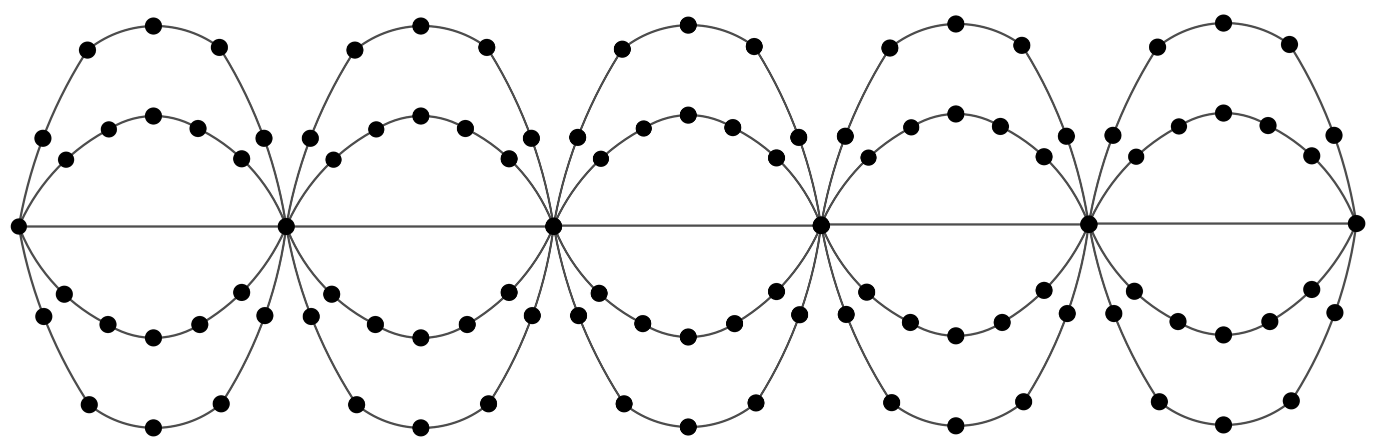

Figure 1. $S(7, 4, 6)$

The rainbow connection number is already known [4] for $G = S(3, k, n)$ with $k \in \{1, 2, 3\}$. In this case,

$$rc(G) = \begin{cases} diam(G) + 1, & \text{if } n = k = 3; \\ diam(G), & \text{otherwise.} \end{cases}$$

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Snake design vector created by freepik - www.freepik.com

Result

Lemma

Let $G = S(\ell, k, n)$.

$$diam(G) = \begin{cases} \lfloor \ell/2 \rfloor, & \text{if } n = 2 \text{ and } k = 1; \\ \ell - 1, & \text{if } n = 2 \text{ and } k > 1; \\ 2\lfloor \ell/2 \rfloor + n - 3, & \text{if } n > 2. \end{cases}$$

Theorem

Let $G = S(\ell, k, n)$.

$$rc(G) \leq \begin{cases} diam(G) + 1, & \text{if } \ell \text{ is even or } n = 2; \\ diam(G) + 2, & \text{if } \ell \text{ is odd.} \end{cases}$$

This bound is near-tight, since we know snake graphs which have $rc(G) = diam(G) + 1$.

Proof (sketch). Fig. 2 shows a rainbow coloring of the *block* $B_{i,i+1}$, for $0 \leq i \leq n-2$.

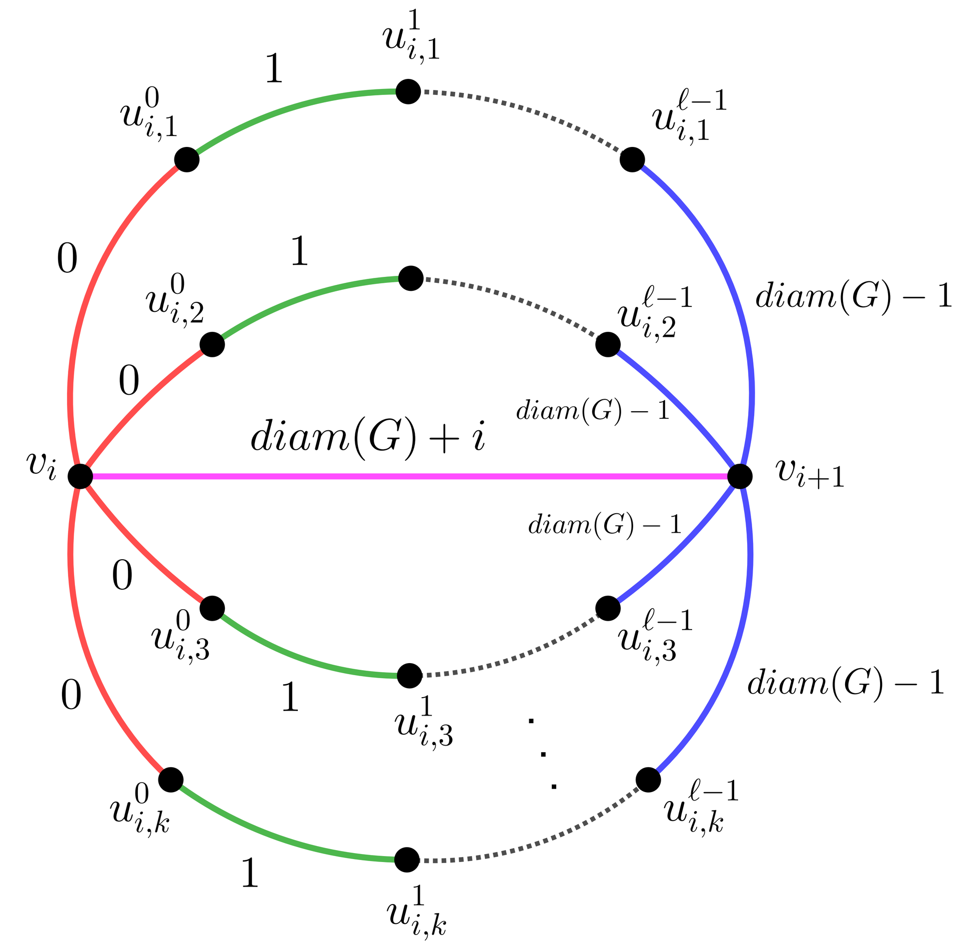

Figure 2. $B_{i,i+1}$

Fig. 3 shows $S(4, 4, 4)$ with the rainbow coloring obtained.

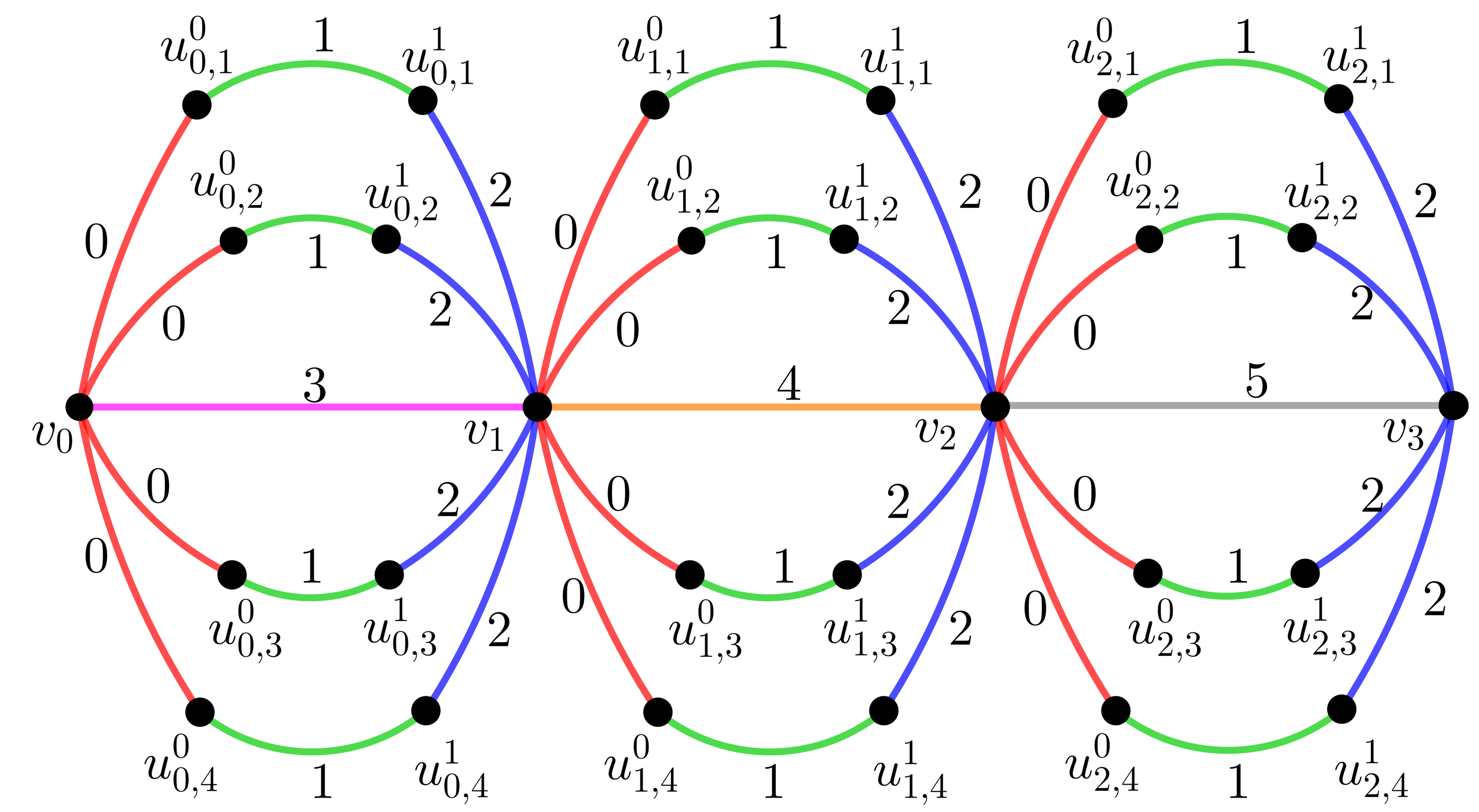

Figure 3. $S(4, 4, 4)$

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