A Near-tight Bound for the Rainbow Connection Number of Snake Graphs

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Introduction

The rainbow connection number of a connected graph $G$, denoted $rc(G)$, is the least $k$ for which $G$ admits a (not necessarily proper) $k$-edge-coloring such that between any pair of vertices there is a path whose edge colors are all distinct. This parameter has important applications [3].

Remark ([1]) If $G$ is a connected and not trivial graph with $n$ vertices, then $diam(G) \leq rc(G) \leq |E(G)|$.

We present a near-tight bound for the rainbow connection number of snake graphs, a class commonly studied in labeling problems [2,5].

Let $\ell \geq 3$, $k \geq 1$, $n \geq 2$. An $\ell$-gon $k$-multiple snake graph over $n$ vertices, denoted $S(\ell, k, n)$, is obtained from $P_n$: $v_0v_1 \ldots v_{n-1}$ by adding $k$ multiple edges between $v_i$ and $v_{i+1}$ for $0 \leq i \leq n - 2$ and making $\ell - 2$ successive subdivisions at each edge added. See Fig. 1.

Result

Lemma

Let $G = S(\ell, k, n)$.

$$diam(G) = \begin{cases} \lfloor \ell/2 \rfloor, & \text{if } n = 2 \text{ and } k = 1; \\ \ell - 1, & \text{if } n = 2 \text{ and } k > 1; \\ 2\lfloor \ell/2 \rfloor + n - 3, & \text{if } n > 2. \end{cases}$$

Theorem

Let $G = S(\ell, k, n)$.

$$rc(G) \leq \begin{cases} diam(G) + 1, & \text{if } \ell \text{ is even or } n = 2; \\ diam(G) + 2, & \text{if } \ell \text{ is odd}. \end{cases}$$

This bound is near-tight, since we know snake graphs which have $rc(G) = diam(G) + 1$.

Proof (sketch). Fig. 2 shows a rainbow coloring of the block $B_{i,i+1}$, for $0 \leq i \leq n - 2$.

References


