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B_1-EPG representations using block-cutpoint trees

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Introduction

EPG graphs were first introduced by Golumbic et al in [2] motivated from circuit layout problems [1]. In B_1-EPG representations, each path has one of the following shapes x = {⌞,⌜,⌟,⌝}, besides horizontal or vertical segments. One may consider more restrictive subclasses of B_1-EPG by limiting the types of bends allowed in the representation, that is, only the paths in a subset of x are allowed.
Ex.: The ⌞⌝-EPG graphs are those in which only the "⌞" or the "⌝" shapes are allowed.

Objective

We show that two superclasses of trees are B_1-EPG (one of them being the cactus graphs). On the other hand, we show that the block graphs are ⌞-EPG and provide a linear time algorithm to produce ⌞-EPG representations of generalization of trees. These proofs employed a new technique from previous results based on block-cutpoint trees of the respective graphs.

Preliminaries

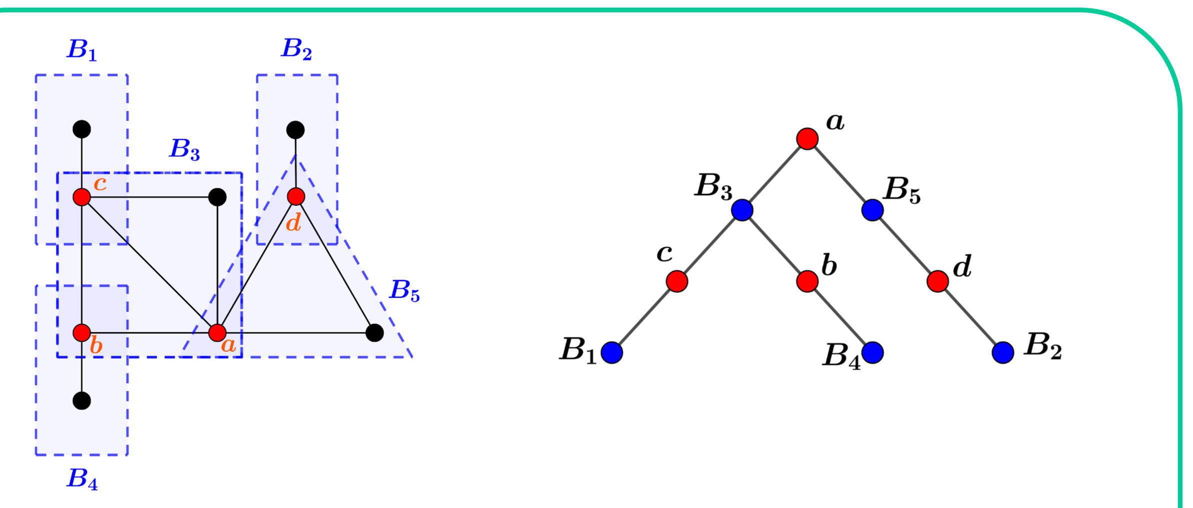

Figure 1: A graph and its respective BC-tree. The cut vertices are marked in red.

Consider a graph G. Let T be a bipartite graph in which the parts X and Y are such that X contains one vertex b for each block B of G, called a *block vertex*, and Y contains one vertex c for each cut vertex c' of G, called as such in T. Vertices b and c form an edge if $c' \in V(B)$. It is easy to see that T is in fact a tree. We define T as the *block-cutpoint tree of G* [3] (BC-tree). See Figure 1.

B1-EPG representations

We describe a B_1-EPG representation of a superclass of trees, inspired on the representation of trees described in [2]. The novelty of our results is the usage of BC-trees to obtain EPG representations, which will be employed to obtain B_1-EPG representations of more general classes of graphs.

Theorem 1

Let G be a graph such that every block of G is B_1-EPG and every cut vertex v of G is a universal vertex in the blocks of G in which v is contained. Then, G is B_1-EPG.

Proof. (Sketch) The theorem is proved by induction. Actually, we prove a stronger claim, stated as follows: given any graph G satisfying the theorem conditions and a BC-tree T of G rooted at some cut vertex r, there exists a B_1-EPG representation $R=\{P_v \mid v \in V(G)\}$ of G in which:

i. P_r is a vertical path with no bends in R;
ii. all paths but P_r are constrained within the horizontal portion of the grid defined by P_r and at the right of it.

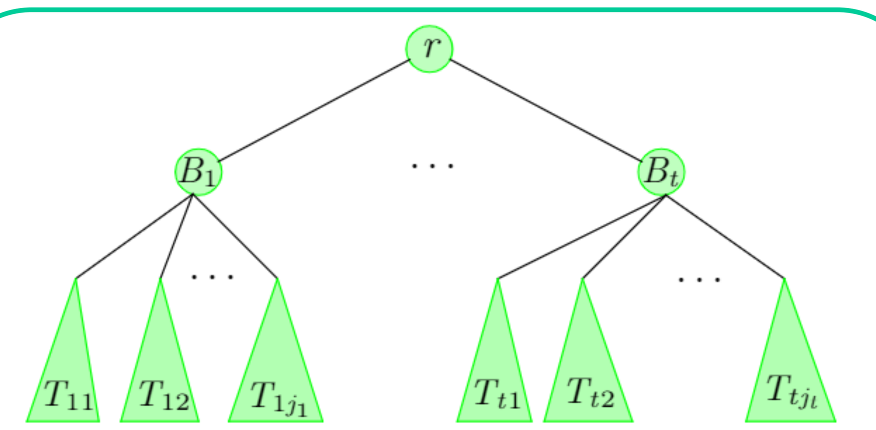

Figure 2: The rooted BC-tree T of a graph.

From T (the BC-tree of G shown in Figure 2), build the representation R of G as follows. First, build an arbitrary vertical path P_r in the grid $\mathcal{G}$, corresponding the root r. Next, divide the vertical portion of $\mathcal{G}$ defined by P_r and at the right of it into t vertical subgrids, $\mathcal{G}_1$, $\mathcal{G}_2$, . . . , $\mathcal{G}_t$, with a row space between them such that the i-th subgrid will contain the paths corresponding to the cut vertices that are descendants of B_i in T. So, each subgrid $\mathcal{G}_i$ is constructed as shown in Figure 3.

We first represent the children of B_i as disjoint ⌞-shaped paths, all sharing the same grid column in which P_r lies. For each B_i, we build the paths in B_i', that correspond to vertices of B_i that are not cut vertices of G (as those in black in Figure 1), and the paths in T_{ij}, belonging to G[T_{ij}], for all $1 \le j \le j_i$. So, it remains to define how the paths belonging to the regions B_i' and T_{ij} will be built.

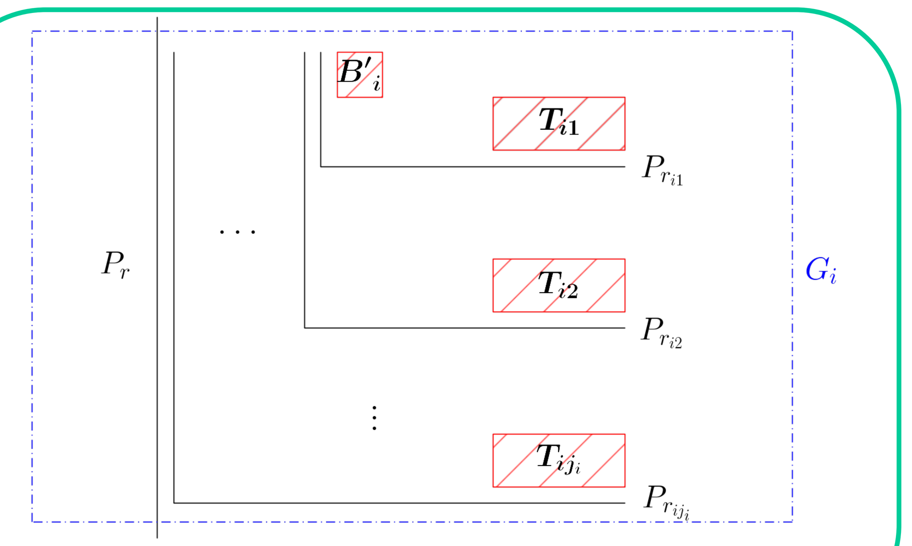

Figure 3: A subgrid $\mathcal{G}_i$.

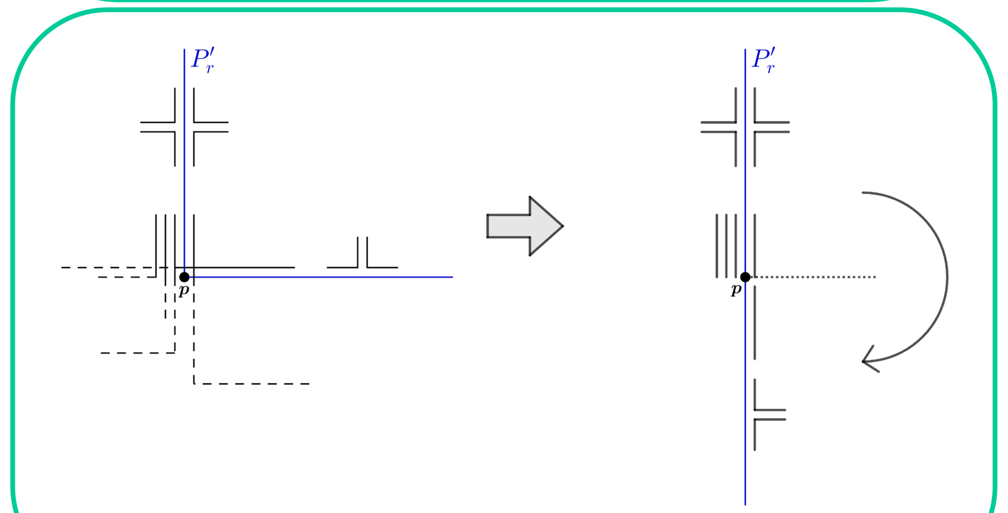

Figure 4: Transforming R'.

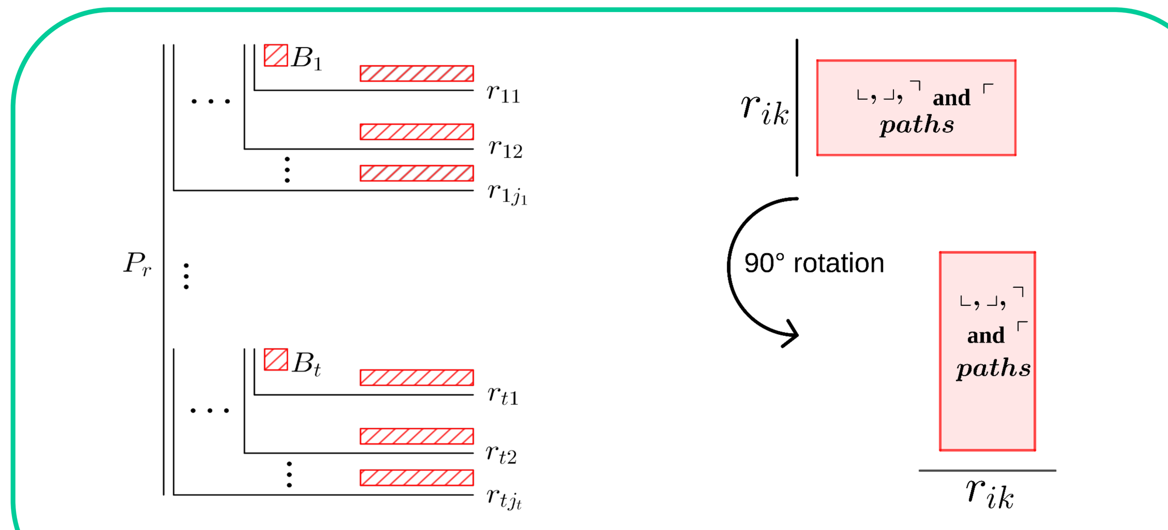

Figure 5: B_1-EPG representation of G after induction step.

Let R' be a B_1-EPG representation of B_i and let P_r' be an ⌞-path corresponding r in R'. Since r is universal to B_i, it is possible to transform R' such that all universal vertices become vertical paths as shown in Figure 4. For the T_{ij} portion of the representation, let r_{ij} be the root of T_{ij}. Applying induction hypothesis, we obtain B_1-EPG representations of each subtree that have vertical paths representing each root and the entire representation is bounded as described previously in (ii).

Thus, we can attach each one of the representations to its respective portion of the model being built, rotated 90 degrees in counter-clockwise (see Figure 5).

Theorem 2

Let G be a graph such that every block of G is ⌞ -EPG and every cut vertex v of G is a universal vertex in the blocks of G in which v is contained. Then, G is B_1-EPG.

Proof. (Sketch) This proof follows the same reasoning lines as those in the proof of Theorem 1. However, the assumption that every block B_i is ⌞-EPG allows their EPG representations to be transformed into interval models. It is possible to show how to build an interval model of each block, given an ⌞-EPG representation of it. Furthermore, the EPG representations of the subtrees T_{ij}, $1 \le j \le j_i$, of B_i, for all i, obtained after the induction step can be transformed into ⌞-EPG models by 90 degree clockwise rotation so that the entire representation is ⌞-EPG.

Theorem 3

Cactus graphs are B_1-EPG

Proof. (Sketch) This proof follows the same reasoning lines as those in the proof of Theorem 1. The difference here is that every block is either an edge or a cycle. It is possible therefore to construct B_1-EPG representations of every block B_i. Furthermore, the B_1-EPG representations of the subtrees T_{ij}, $1 \le j \le j_i$, of B_i, for all i, obtained after the induction step can be shown possible to be attached into vertical or horizontal regions of the cycle/edge so that the entire representation is B_1-EPG.

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Acknowledgment

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