B₁-EPG representations using block-cutpoint trees

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Introduction

EPG graphs were first introduced by Golumbic et al in [2] motivated from circuit layout problems [1]. In B₁-EPG representations, each path has one of the following shapes s = (r, s, r), besides horizontal or vertical segments. One may consider more restrictive subclasses of B₁-EPG by limiting the types of bends allowed in the representation, that is, only the paths in a subset of s are allowed. Ex.: The GEP graphs are those in which only the “<” or the “>” shapes are allowed.

Objective

We show that two superclasses of trees are B₁-EPG (one of them being the cactus graphs). On the other hand, we show that the block graphs are ω-EPG and provide a linear time algorithm to produce ω-EPG representations of generalization of trees. These proofs employed a new technique from previous results based on block-cutpoint trees of the respective graphs.

Preliminaries

Consider a graph G. Let T be a bipartite graph in which the parts X and Y are such that X contains one vertex b for each block B of G, called a block vertex, and Y contains one vertex c for each cut vertex c' of G, called as such in T. Vertices b and c form an edge if c' ∈ V(B). It is easy to see that T is in fact a tree. We define T as the block-cutpoint tree of G [3] (BC-tree). See Figure 1.

B₁-EPG representations

We describe a B₁-EPG representation of a superclass of trees, inspired on the representation of trees described in [2]. The novelty of our results is the usage of BC-trees to obtain EPG representations, which will be employed to obtain B₁-EPG representations of more general classes of graphs.

Theorem 1

Let G be a graph such that every block of G is B₁-EPG and every cut vertex v of G is a universal vertex in the blocks of G in which v is contained. Then, G is B₁-EPG.

Proof (Sketch) The theorem is proved by induction. Actually, we prove a stronger claim, stated as follows: given any graph G satisfying the theorem conditions and a BC-tree T of G rooted at some cut vertex r, there exists a B₁-EPG representation R = {P_v | v ∈ V(G)} of G in which:

i. P_r is a vertical path with no bends in R;
ii. all paths but P_r are constrained within the horizontal portion of the grid defined by P_r and at the right of it.

From T (the BC-tree of G shown in Figure 2), build the representation R of G as follows. First, build an arbitrary vertical path P_r in the grid G corresponding the root r. Next, divide the vertical portion of G defined by P_r, and at the right of it into r vertical subgrids, G_1, G_2, ..., G_v, with a row space between them such that the i-th subgrid will contain the paths corresponding to the cut vertices that are descendants of B_i in T. So, each subgrid G_i is constructed as shown in Figure 3.

We first represent the children of B_i as disjoint L-shaped paths, all sharing the same grid column in which P_r lies. For each B_i, we build the paths in B_i, that correspond to vertices of B_i that are not cut vertices of G (as those in black in Figure 1), and the paths in T_{B_i} belonging to G[T_{B_i}], for all 1 ≤ j ≤ j_i.

Thus, we can attach each one of the representations to its respective portion of the model being built, rotated 90 degrees in counter-clockwise (see Figure 5).

Theorem 2

Let G be a graph such that every block of G is ω-EPG and every cut vertex v of G is a universal vertex in the blocks of G in which v is contained. Then, G is B₁-EPG.

Proof (Sketch) This proof follows the same reasoning lines as those in the proof of Theorem 1. However, the assumption that every block B_i is ω-EPG allows their EPG representations to be transformed into interval models. It is possible to show how to build an interval model of each block, given an ω-EPG representation of it. Furthermore, the EPG representations of the subtrees T_{B_i} 1 ≤ j ≤ j_i of B_i, for all i, obtained after the induction step can be transformed into ω-EPG models by 90 degree clockwise rotation so that the entire representation is ω-EPG.

Theorem 3

Cactus graphs are B₁-EPG

Proof. (Sketch) This proof follows the same reasoning lines as those in the proof of Theorem 1. The difference here is that every block is either an edge or a cycle. It is possible therefore to construct B₁-EPG representations of every block B_i. Furthermore, the B₁-EPG representations of the subtrees T_{B_i} 1 ≤ j ≤ j_i of B_i, for all i, obtained after the induction step can be shown possible to be attached into vertical or horizontal regions of the cycle/edge so that the entire representation is B₁-EPG.

References


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