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Alliance and Domination on Uniformly Clique-expanded Graphs

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Introduction

This work aims at presenting the uniformly clique-expanded graphs and its results on global defensive alliance and total dominating set problems. Those graphs are related to Sierpiński graphs [5] and subdivided-line graphs [1]. We show the minimum cardinality of the global defensive alliance for some particular situations of uniformly clique-expanded graphs, and we also relate that cardinality to the total dominating set number for graphs having a path or cycle as the root.

Basic Definitions

Consider $G = (V, E)$ a finite, simple, and undirected graph. We write P_n, C_n, and K_n for a *path*, *cycle*, and *clique* of the order n, resp. For the *closed* (resp. *open*) *neighborhood of a vertex* $v \in V$, we denote it by $N[v]$ (resp. $N(v)$). Analogously, we use $N[S]$ (resp. $N(S)$) for the *closed* (resp. *open*) *neighborhood of a vertex subset* $S \subseteq V$. A vertex subset $S \subseteq V$ is said a *dominating set* if $N[S] = V$. Moreover, we call the subset S by *total dominating* set only for $N(S) = V$. Now, S is a *defensive alliance* if it satisfies $|N[v] \cap S| \geq |N(v) \cap (V/S)|$ for every $v \in S$. When S is both a defensive alliance and a dominating set, we say S is a *global defensive alliance*. We denote $\gamma_t(G)$ (and $\gamma_a(G)$) as the minimum cardinality of a total dominating set (and global defensive alliance) of G.

The Main Definition & an Example

We say that a graph H is a *uniformly clique-expanded graph* if there exist a graph G and a clique K_n with $n \geq \Delta(G)$ (maximum degree of G) satisfying: (1) $V(H)$ consists of vertices from K_n^v, which is a copy of the clique K_n, for each vertex v of G, and (2) $E(H)$ contains edges of all clique copies, and every edge $(u)(v)$ linking a vertex $(u) \in K_n^u$ to some $(v) \in K_n^v$ since $uv \in E(G)$ and no edges coincide end-vertices in H besides the ones inside of cliques. G is the so-called *root* of H. See an example in Figure 1.

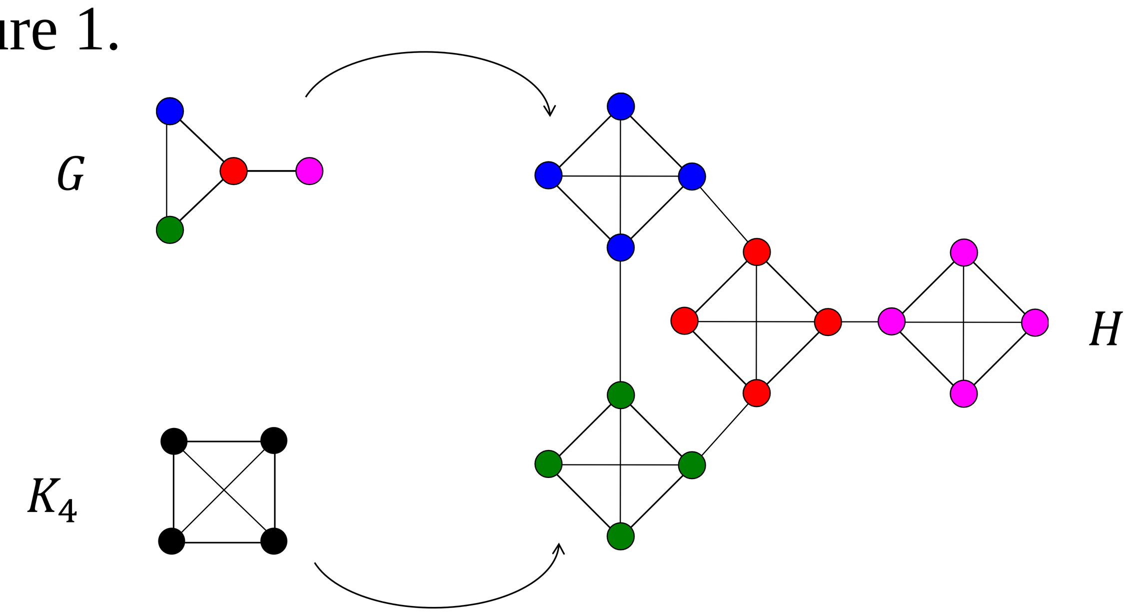

Figure 1: The graph H can be obtained from the root G and the clique K_4, and so it is a uniformly clique-expanded graph.

Results

Theorem 1: Let H be a uniformly clique-expanded graph from a root G and a clique K_n. If n is even and $\Delta(G) \leq \frac{n}{2}$, then $\gamma_a(H) = \frac{n}{2}|V(G)|$.

Theorem 2: Let H be a uniformly clique-expanded graph from a root G and a clique K_n. If n is odd and $\Delta(G) \leq \frac{n-1}{2}$, then: $\gamma_a(H) = \sum_{d(v)<\frac{n-1}{2}} \frac{n+1}{2} + \sum_{d(v)=\frac{n-1}{2}} \frac{n-1}{2}$, for all $u \in V(G)$, where $d(u)$ is the degree of v in G.

Now, the next theorem arises from properties in [2,3,4].

Theorem 3: Let H be a uniformly clique-expanded graph from a root $G \in \{P_q, C_q\}$, $q \geq 2$, and a clique K_n. We have $\gamma_t(H) = q + q \bmod 2$, and if:

i. G is a cycle and:
- *a.* $2 \leq n \leq 3$, then $\gamma_a(H) = \gamma_t(H)$;
- *b.* $4 \leq n \leq 5$, then $\gamma_a(H) = \lfloor \frac{n}{2} \rfloor q$;
- *c.* $n \geq 6$, then $\gamma_a(H) = \lceil \frac{n}{2} \rceil q$.

ii. G is a path and:
- *a.* $n = 2$, then $\gamma_a(H) = \gamma_t(H) - 1$ whether $p \equiv 1 (\bmod 2)$ or $\gamma_a(H) = \gamma_t(H)$ otherwise;
- *b.* $n = 3$, then, $\gamma_a(H) = \gamma_t(H)$;
- *c.* $n = 4$, then $\gamma_a(H) = \frac{n}{2} q$.
- *d.* $n = 5$, then $\gamma_a(H) = \frac{n-1}{2} q$.
- *e.* $n \geq 6$, then $\gamma_a(H) = \lceil \frac{n}{2} \rceil q$.

Conclusions & Remarks

The uniformly clique-expanded graphs are particular cases of line graphs of bipartite graphs since we can verify that they are (claw,diamond,odd-hole)-free. Thus, we presented preliminary results that somehow are important to the well-known superclass.

References

[1] HASUNUMA T. Structural Properties of Subdivided-Line Graphs. In: Lecroq T., Mouchard L. (eds) Combinatorial Algorithms. IWOCA 2013. Lecture Notes in Computer Science, vol 8288. Springer, Berlin, Heidelberg, 2013. https://doi.org/10.1007/978-3-642-45278-9_19

[2] HAYNES, T. W.; HEDETNIEMI, S.T.; HENNING, M.A. Global defensive alliances in graphs. The Electronic Journal of Combinatorics, 2003.

[3] HENNING, M.A.: Graphs with large total domination number. J. Graph Theory 563 35(1), 21–45 (Sep 2000)

[4] KAHINA OUAZINE, HACHEM SLIMANI, ABDELKAMEL TARI. Alliances in graphs: Parameters, properties and applications – A survey. AKCE International Journal of Graphs and Combinatorics. DOI: 10.1016/j.akcej.2017.05.002, 2018.

[5] KLAVŽAR, S., MILUTINOVIĆ, U., PETR, C. 1-perfect codes in sierpiński graphs. Bulletin of the Australian Mathematical Society66(3), 369–384. (2002)

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