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Kernelization lower bounds for Multicolored Independent Set

Guilherme C. M. Gomes and Vinicius F. dos Santos

Departamento de Ciência da Computação — Universidade Federal de Minas Gerais
{gcm.gomes, viniciussantos}@dcc.ufmg.br

1. Introduction

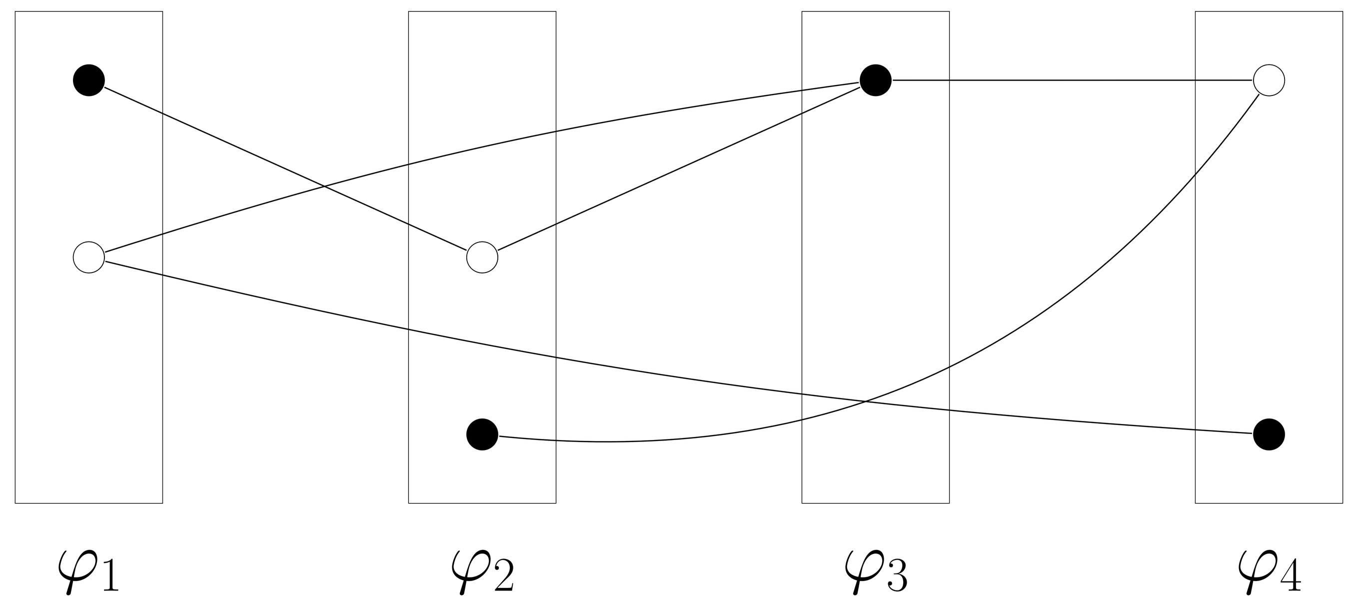
Pre-processing algorithms are frequently employed when solving large problems and are often fundamental to do so. Until recently, however, these algorithms were designed without theoretical guarantees, and measuring their effectiveness was a completely empirical process. Parameterized complexity offers a sound theoretical framework that allows us to prove lower and upper bounds for these **kernelization** algorithms, as they came to be known in the community [2]. Given an instance (x, k) of a parameterized problem Π, we say that Π admits a kernel of size $g(k)$ when parameterized by k if we can build an equivalent Π instance of size at most $g(k)$. Motivated by the fact that Multicolored Independent Set is a central problem in parameterized complexity, we prove the following theorem, where a class $\mathcal{G}$ is non-trivial if, for every $t \in \mathbb{N}$, $\mathcal{G}$ contains a graph on t vertices; we point out that Independent Set does admit a polynomial kernel [3] under vertex cover.

2. The theorem

For every fixed non-trivial graph class $\mathcal{G}$, Multicolored Independent Set does not admit a polynomial kernel when jointly parameterized by vertex deletion distance to $\mathcal{G}$ and size of the solution, unless $\mathsf{NP} \subseteq \mathsf{coNP/poly}$.

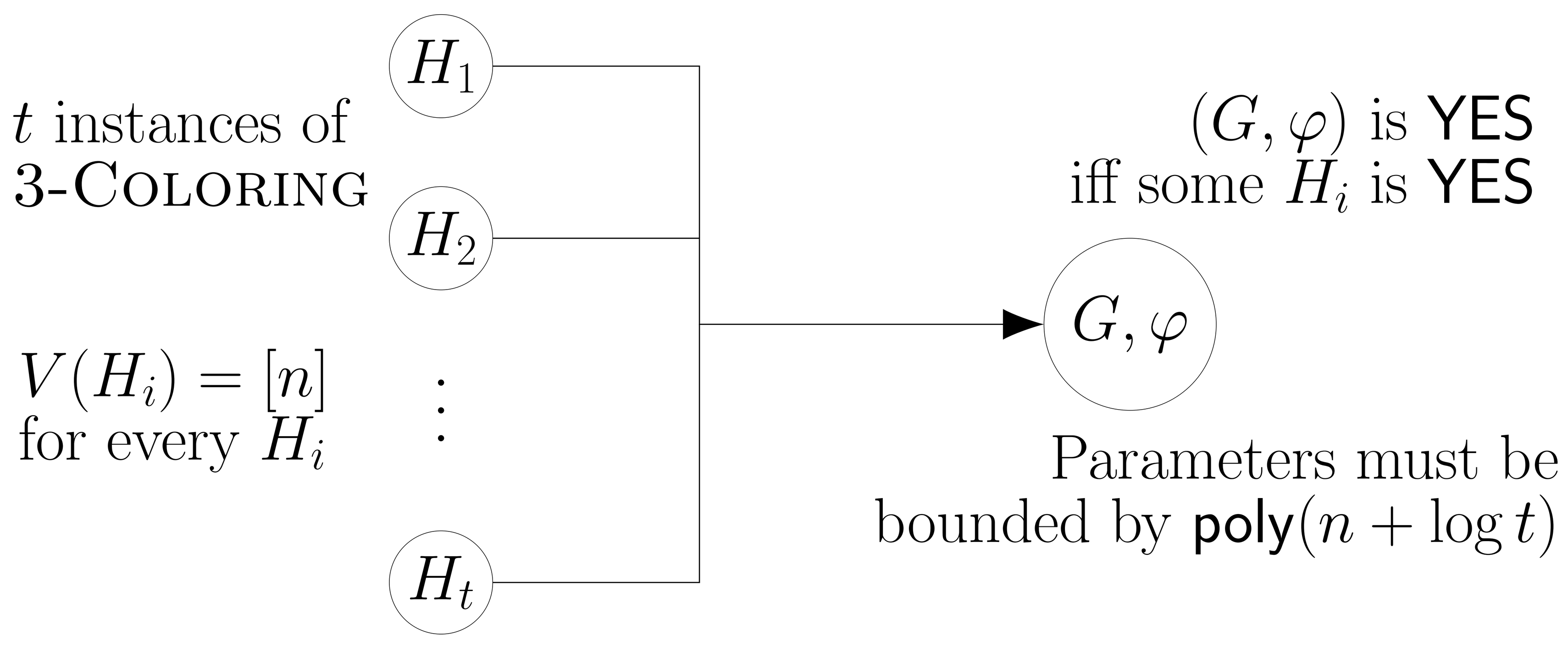
3. Multicolored Independent Set

An instance is a pair (G, φ) where G is a graph, φ is a partition of $V(G)$, and the goal is to find an independent set of G that hits each part of φ exactly once.



4. Cross-composition

We use the cross-composition framework of Bodlaender et al. [1] to show that 3-Coloring OR-cross-composes into Multicolored Independent Set parameterized by distance to $\mathcal{G}$ and size of the solution. That is, it is a many to one reduction with the following constraints:

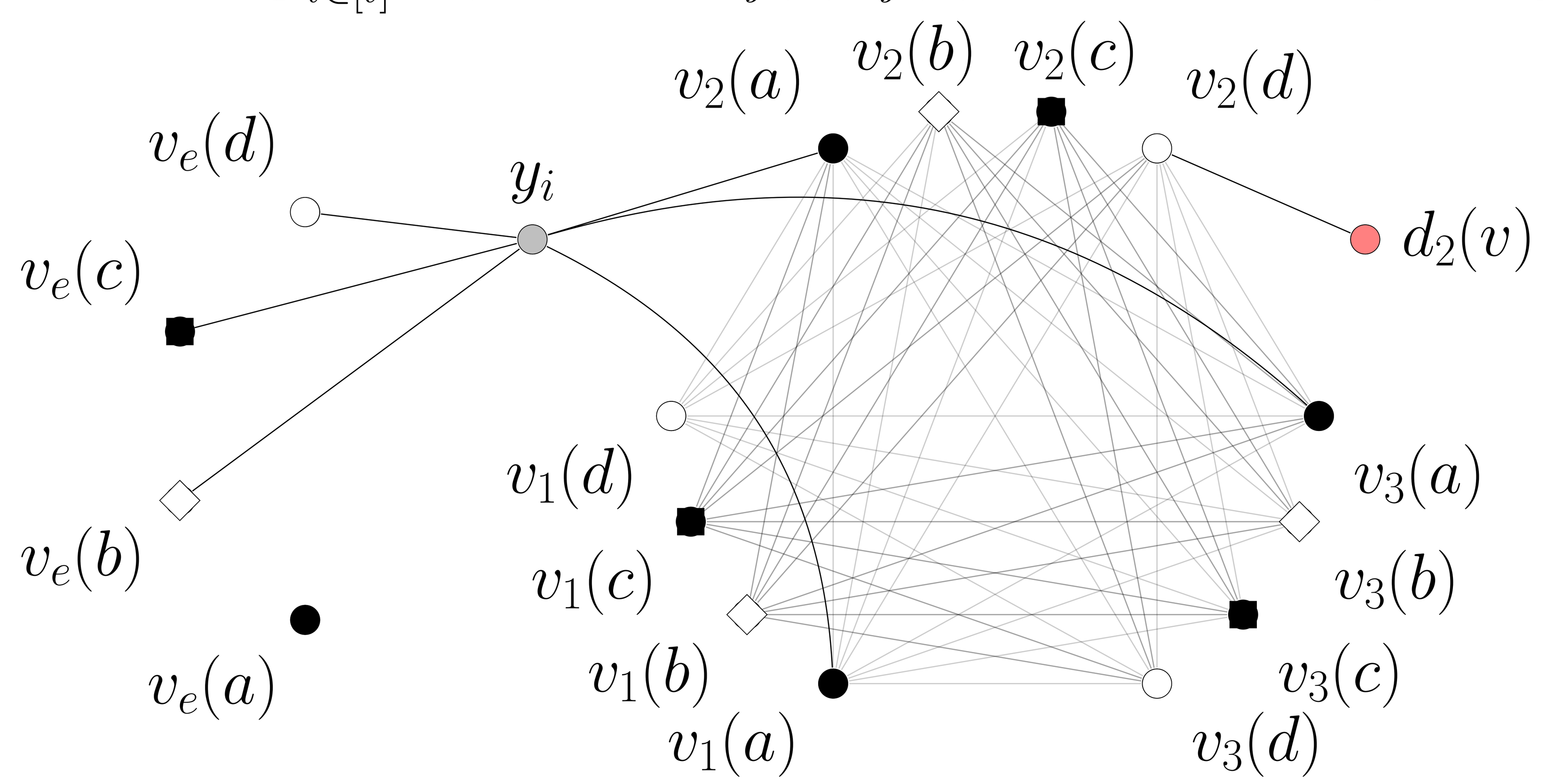


5. Instance Selector Gadget

Begin by adding to G a set $Y = \{y_1, \dots, y_t\}$ that induces a graph of $\mathcal{G}$, and add Y as a part of φ.

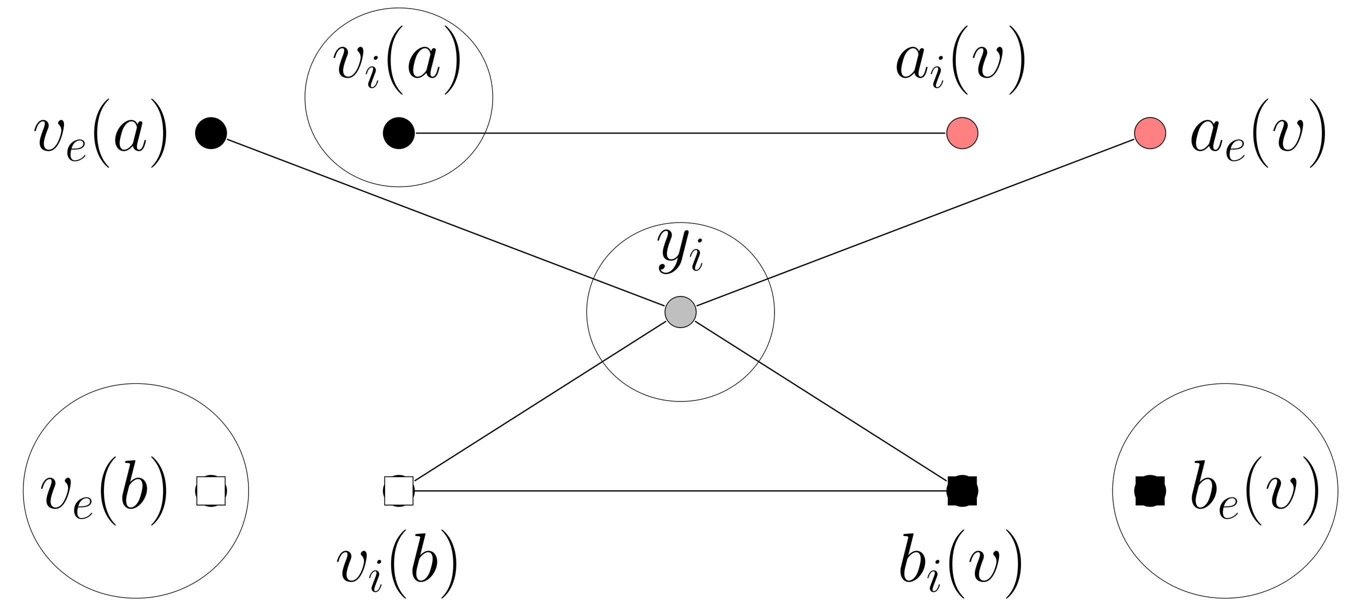
6. Vertex Gadget

For each $v \in [n]$, add to G a gadget G_v containing an independent set $A(v) = \{v_e(a) \mid a \in [n] \setminus \{v\}\}$ and a copy $K(v)$ of the complete tripartite graph $K_{n-1,n-1,n-1}$. For each G_v, add parts $\{p(v, a) \mid a \in [n] \setminus \{v\}\}$ to φ; each $p(v, a)$ contains $v_e(a)$ and three non-adjacent vertices $v_1(a), v_2(a), v_3(a)$ of $K(v)$. For each H_i, if $av \notin E(H_i)$, add edges $\{y_i v_j(a)\}_{j\in[3]}$ and $\{y_i a_j(v)\}_{j\in[3]}$, otherwise add edges $y_i v_e(a)$ and $y_i a_e(v)$. For each $av \in \bigcup_{i\in[t]} E(H_i)$, add $v_j(a)a_j(v)$ for every $j \in \{1, 2, 3\}$.



7. Intuition

- For each $v \in [n]$ we can only choose vertices of one color class of $K(v) \Rightarrow v_i(\cdot)$ is in the solution $\mathcal{I}$ if and only if we color $v \in [n]$ with color i.
- If $v_i(a) \in \mathcal{I}$, then $a_i(v) \notin \mathcal{I} \Rightarrow v$ and a cannot have the same color.
- If $v_e(a) \in \mathcal{I}$, we can ignore edge $av \Rightarrow$ when $v_e(a) \in \mathcal{I}$, v and a can have the same color.
- There is a unique $y_i \in \mathcal{I}$ and, for every $av \in E(H_i)$, $v_e(a) \notin \mathcal{I}$, so some $v_i(a)$ must be in $\mathcal{I}$ and $a_i(v)$ must not $\Rightarrow$ if $y_i \in \mathcal{I}$, vertices that are adjacent in H_i cannot have the same color.



References

[1] Hans L. Bodlaender, Bart M. P. Jansen, and Stefan Kratsch. "Cross-Composition: A New Technique for Kernelization Lower Bounds". In: *Proc. of the 28th International Symposium on Theoretical Aspects of Computer Science (STACS)*. Vol. 9. LIPIcs. 2011, pp. 165–176.

[2] Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Daniel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms*. 1st. Springer Publishing Company, Incorporated, 2015. isbn: 3319212745.

[3] Fedor V. Fomin, Bart M.P. Jansen, and Michał Pilipczuk. "Preprocessing subgraph and minor problems: When does a small vertex cover help?" In: *Journal of Computer and System Sciences* 80.2 (2014), pp. 468 –495. issn: 0022-0000.