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Some results on Vertex Separator Reconfiguration

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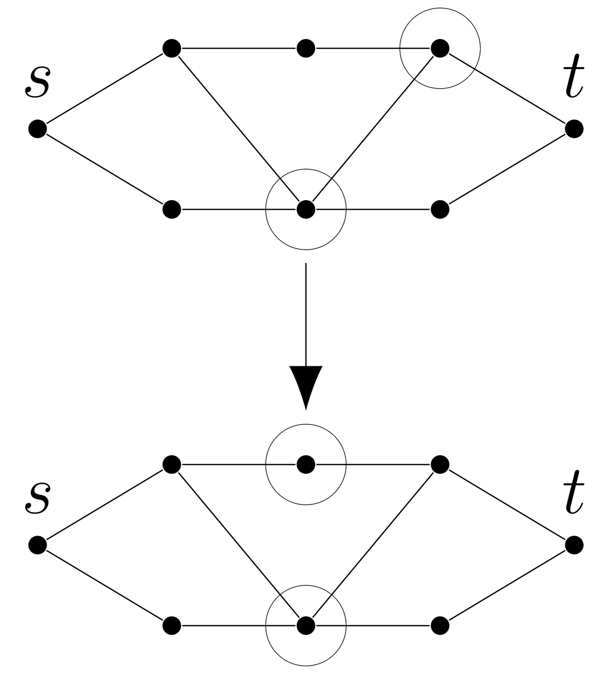
1. Introduction

Graph reconfiguration problems have been studied extensively in the literature, with Independent Set Reconfiguration [3] being by far the favorite research topic. Nevertheless, reconfiguration problems of other graph structures, such as vertex covers [4] and vertex colorings [1], have also been investigated. No previous work, however, has dealt with the reconfiguration of vertex separators. In this work, we begin this study in the form of the Vertex Separator Reconfiguration problem. In this problem, we are given a graph G and two st-separators S_a and S_b of G, and the goal is to reconfigure S_a into S_b. We prove its complexity on a subclass of bipartite graph under the three most common reconfiguration rules: token sliding (TS), token jumping (TJ), and token addition/removal (TAR); being **PSPACE-hard** under TS and **NP-hard** under the other two. We also show that TS and TAR computationally equivalent.

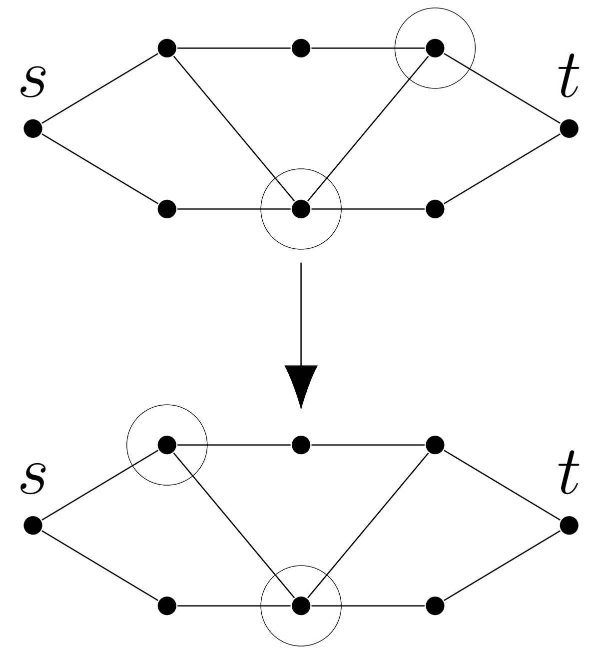
2. Token Sliding

Replace $v \in A$ with some vertex in $N(v) \setminus A$.



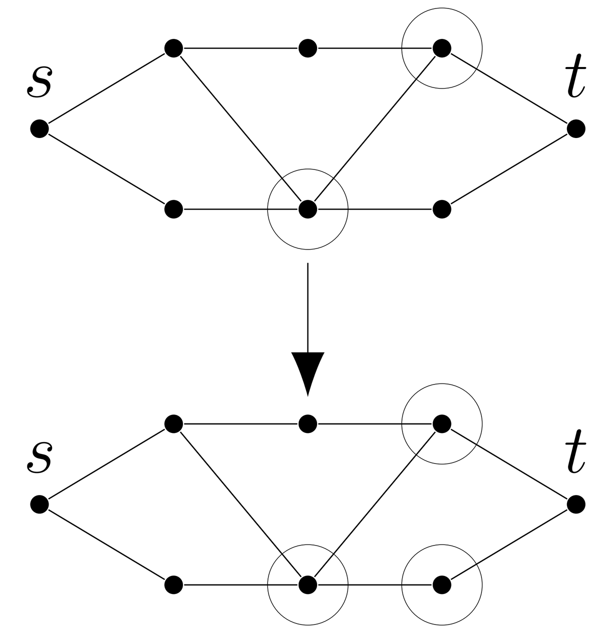
3. Token Jumping

Replace a vertex of A with any other vertex of G not in A.



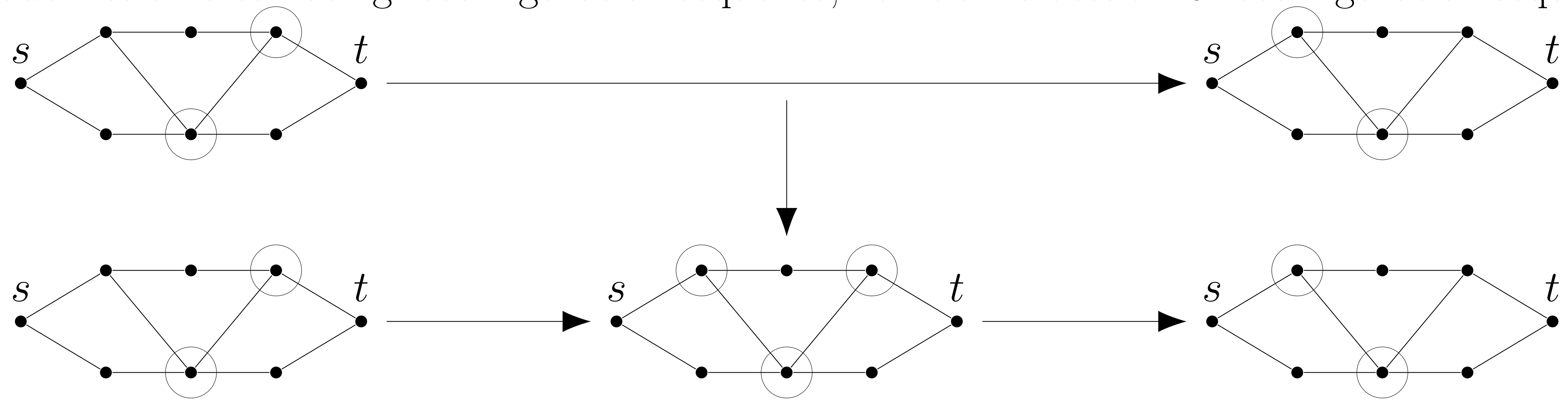
4. k-Token Add./Rem.

Add/remove a vertex from A, so long as the resulting A' satisfies $|A'| \leq k$.



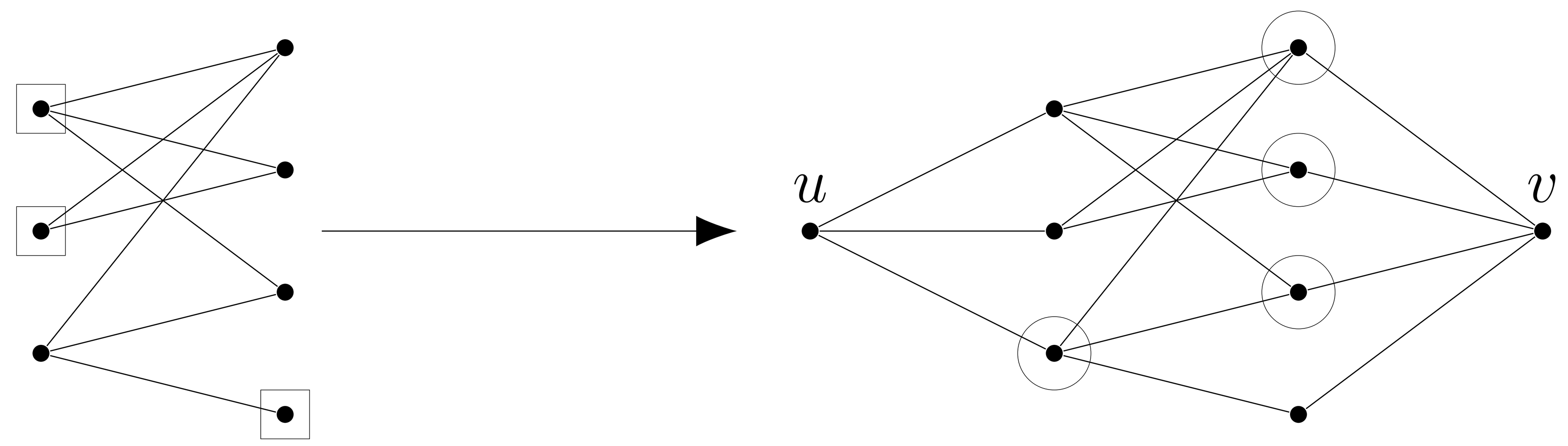
5. TAR/TJ are equivalent

Let us assume that $|S_b| \geq |S_a|$ and $S_a \neq S_b$. We can easily simulate a TJ instance (G, S_a, S_b): just create the TAR instance $(G, S_a, S_b, k+1)$, where $k = \max\{|S_a|, |S_b|\}$. For the converse, given a TAR instance (G, S_a, S_b, k), if $|S_b| = k$ and S_b is minimal, then we answer negatively. Otherwise, pick any two st-separators $S'_a \subseteq S_a$ and $S'_b \subseteq S_b$ of same size and with at most $k-1$ vertices; it follows that (G, S_a, S_b, k) is equivalent to (G, S'_a, S'_b, k) and that we can reconfigure S'_a and S'_b into S_a and S_b, respectively. We can also show that any reconfiguration sequence between S'_a and S'_b can be made into an alternating reconfiguration sequence, i.e. it simulates a TJ reconfiguration sequence.



6. Complexity on bipartite graphs

Let G be a bipartite graph with bipartition A, B and H the bipartite graph obtained by adding to G two vertices u, v such that u is adjacent to every vertex of A and v to every vertex of B. Our reduction is from Independent Set Reconfiguration which is **NP-complete** on bipartite graphs under TJ and **PSPACE-hard** under TS [3]. Its correctness follows from a simple but powerful observation: *A set $I \subseteq V(G)$ is independent in G if and only if $V(G) \setminus I$ is an uv-separator of H.* Formally, if (G, I_a, I_b) is the Independent Set Reconfiguration instance, we construct the equivalent Vertex Separator Reconfiguration $(H, V(G) \setminus I_a, V(G) \setminus I_b)$ where H is defined as before.



7. Final Remarks

We investigated the complexity of the reconfiguration of vertex separators under three commonly studied reconfiguration rules. We also showed that TAR and TJ are computationally equivalent. In the arXiv version of this work [2], we also presented polynomial time algorithms for various classes, including series-parallel graphs and graphs with a polynomial number of minimal separators, which have been omitted here.

References

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[4] Daniel Lokshtanov and Amer E. Mouawad. "The Complexity of Independent Set Reconfiguration on Bipartite Graphs". In: *ACM Trans. Algorithms* 15.1 (Oct. 2018). issn: 1549-6325. doi: `10.1145/3280825`.