On the edge-P_3 and edge-P_3^*-convexity of grid graphs

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Introduction

Consider a graph $G = (V, E)$ and a subset C of V. The *P_3-convex hull* (resp. *P_3^*-convex hull*) of C is obtained by iteratively adding vertices with at least two neighbors in C (resp. two non-adjacent neighbors in C). A subset S of V is *P_3-Helly-independent* (resp. *P_3^*-Helly-independent*) when the intersection of the P_3-convex hulls (resp. P_3^*-convex hulls) of $S \setminus \{v\}$ ($\forall$ $v \in S$) is empty. The *P_3-Helly number* (resp. *P_3^*-Helly number*) is the size of a maximum P_3-Helly-independent (resp. P_3^*-Helly-independent).

The *line graph* L(G) of a graph G is the intersection graph of the edges of G, i.e.,$V(L(G)) = E(G)$ and there is an edge between two vertices in $L(G)$ if the edges they represent in G share a common endpoint. The edge counterparts of P_3-Helly-independent and P_3^*-Helly-independent of a graph follow the same restrictions applied to its edges instead of its vertices, i.e., the edge P_3-convexity (resp. edge P_3^*-convexity) of a graph G is described by P_3-convexity (resp. P_3^*-convexity) of its line graph $L(G)$.

Each vertex of a *grid graph Gpxq* are related to a pair (x, y) that defines its position on the grid with $1 \leqslant x \leqslant p$, $1 \leqslant y \leqslant q$. There is an edge between two vertices of a grid graph if they share a same coordinate x (or y) and the other coordinate differ only by one unit.

In this work, we established the edge P_3^*-Helly number of grid graphs *Gpxq* when both p and q are equal or larger than *16*. Moreover, we give partial results on forbidden configurations of the edge P_3-Helly independent sets of these grid graphs.

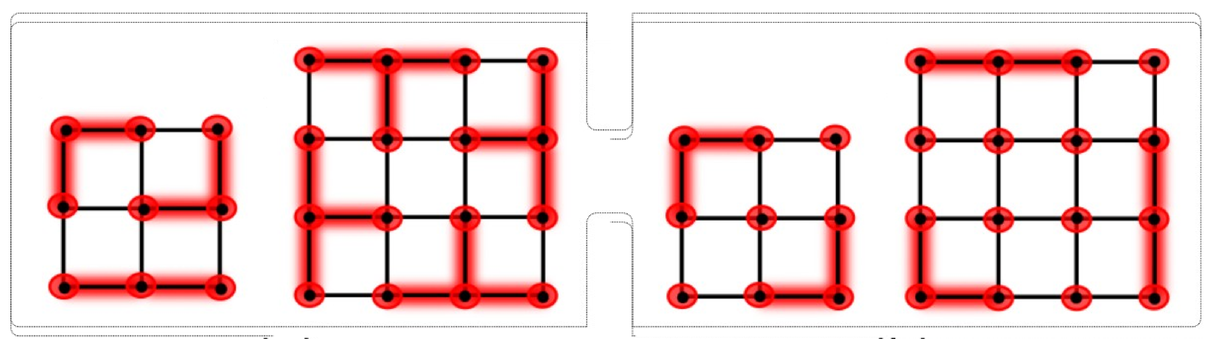
(a) (b)

Figure 1: The red edges are an (a) edge P_3^*-Helly independent (resp. (b) edge P_3-Helly-independent) set of a G3x3 and a G4x4 grid graphs

Motivations and Related Works

There are many applications for graph convexity on distributed systems[7], social networks, and marketing strategies[6]. Moreover, the excelent survey [5] describes several results of the Helly property on graphs. The problem we address considers the Helly property on the edge P_3-convexity and edge P_3^*-convexity of grid graphs.

The first results about the P_3-Helly number on grid graphs appeared in [1]. Later, several results about P_3-Helly number, P_3^*-Helly number and their edge counterparts were established in [2] and [3].

Results

In [3] the authors established that the edge P_3^*-Helly number of a graph occurs between $|V(G)|- \iota (G)$ and $|V(G)|-\gamma (G)$, where $\iota (G)$ is the minimum independent dominating set and $\gamma (G)$ is the minimum dominating set of a graph G.

For grid graphs *Gpxq* with $p \geq 16$ and $q \geq 16$, we have that $\iota (G) = \gamma (G)$. Moreover, the value of $\gamma (G)$ is given by $\lfloor (p+2)(q+2)/5 \rfloor -4$ [4]. Therefore, we know the P_3^*-Helly number of grid graphs *Gpxq* with $p \geq 16$ and $q \geq 16$. The other values for the cases when $p < 16$ and $q < 16$ are computationally obtained. Now, we aim to use these values to obtain the edge P_3^*-Helly numbers when $p < 16$ and $q \geq 16$ or $p \geq 16$ and $q < 16$.

We also consider the edge P_3-Helly independent of grid graphs by establishing several forbidden configurations that allow us to reduce the patterns of possible optimal solutions.

Note that in both cases (a) and (b), the edge P_3 convex hull of the red edges without the horizontal edge contains this edge.

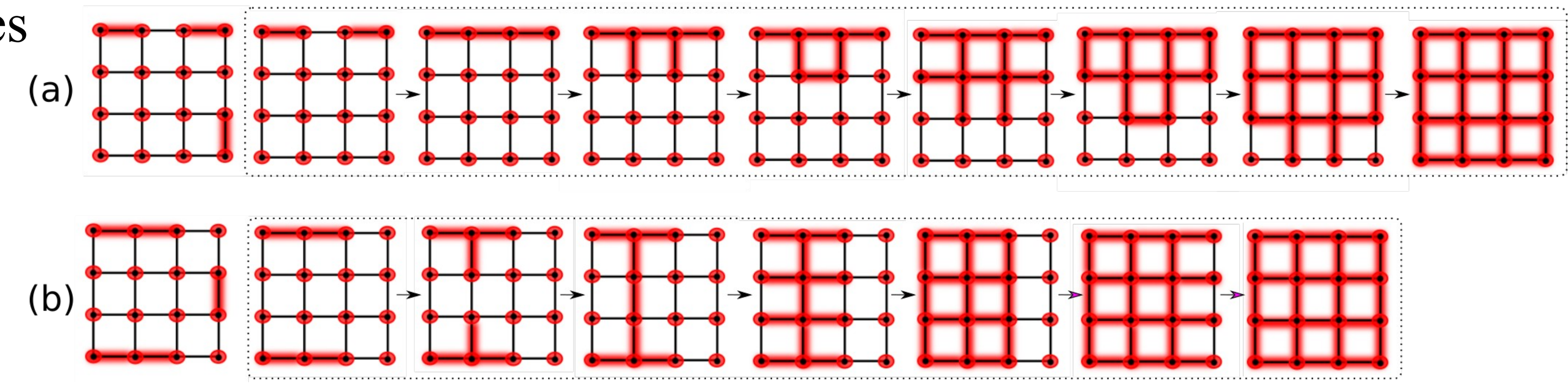

Figure 2: Two forbidden configurations for edge P_3-Helly independent sets.

Ongoing works

We are currently trying to establish the edge P_3-Helly number of grid graphs. Moreover, we want to extend this study to include the other two types of regular grids: the triangular grids and the hexagonal grids.

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