Recognition of Biclique Graphs: What we know so far

Marina Groshaus, André Guedes
Universidade Tecnológica Federal do Paraná (UTFPR)
Universidade Federal do Paraná (UFPR)
E-mails: marinagroshaus@utfpr.edu.br, andre@inf.ufpr.br

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Abstract

The recognition of biclique graphs in general is still open. In recent years we presented some result on the characterization of biclique graphs of graphs of certain graph classes, along with the complexity associated to the recognition problem. Those results introduced some intermediate operators, which we call now as "functors". In this work we summarize all those results and organize the different approaches using the functors.

Introduction

The *biclique graph* of a graph G, denoted by $KB(G)$, is the intersection graph of the bicliques of G. The biclique graph was introduced by Groshaus and Szwarcfiter [8], based on the concept of clique graphs. They gave a characterization of biclique graphs (in general) and a characterization of biclique graphs of bipartite graphs. The time complexity of the problem of recognizing biclique graphs remains open.

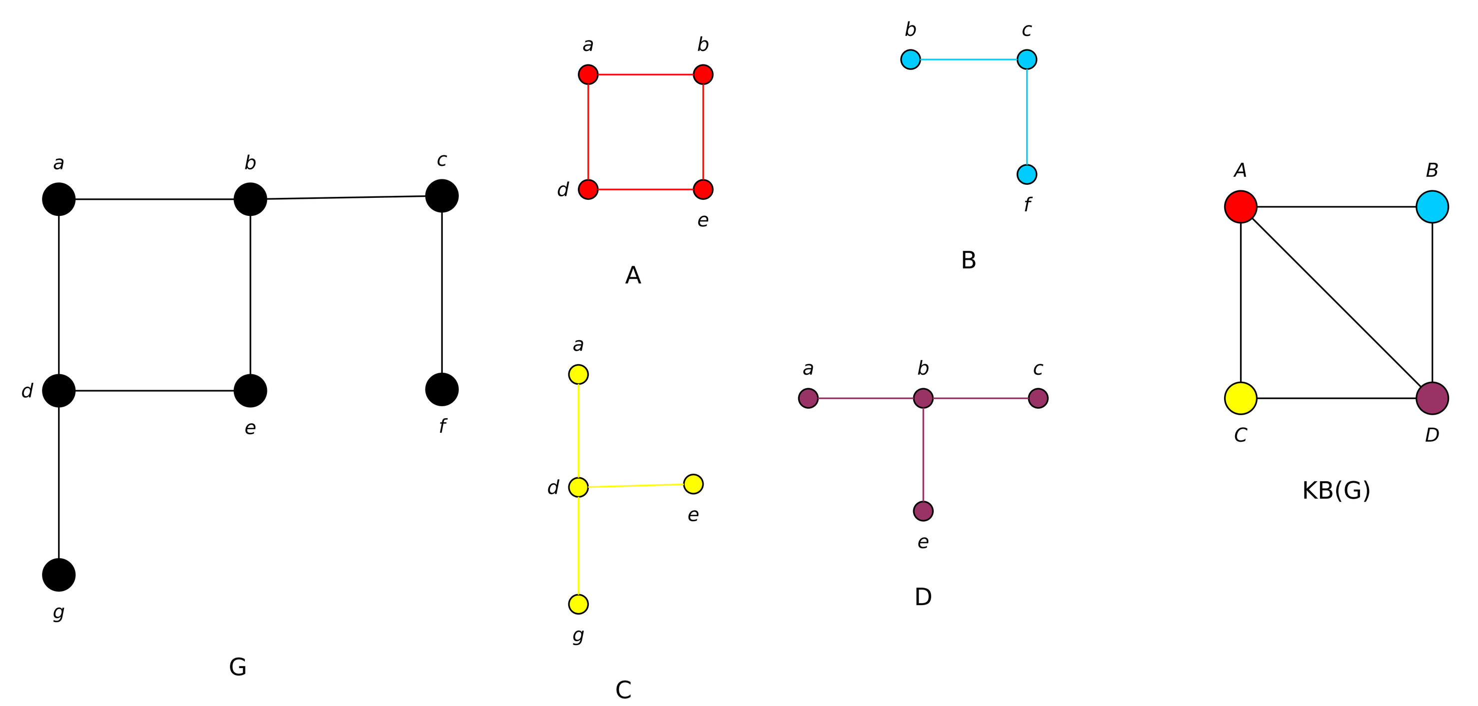

Figure 1: Example of a biclique graph.

Bicliques in graphs have applications in various fields, for example, biology: protein-protein interaction networks, social networks: web community discovery, genetics, medicine, information theory. More applications (including some of these) can be found in the work of Liu, Sim, and Li [10].

The efforts since the definition of the problem of recognizing biclique graphs, similarly to what have been done for others graph operators, are mainly focused on understanding the class $KB(\mathcal{A})$, for some graph class $\mathcal{A}$.

In this work we summarize what is known about recognition of $KB(\mathcal{A})$ for a collection of graph classes.

Classes Studied

- $\mathcal{G}$: All graphs
- $\mathcal{G}_k$: Graphs with girth at least k
- P_n: Path with n vertices
- C_n: Cycle with n vertices
- K_n: Complete graph of order n
- co-CG: Co-comparability graphs
- IIC-comparability: Interval intersection closed comparability graphs [6, 7]
- IIC-PG: IIC-Permutation Graphs = IIC-comparability $\cap$ co-CG [6, 7]
- IBG: Interval bigraphs
- HIB: Helly interval bigraphs [4]
- PIB: Proper interval bigraphs
- PIB-ASG: Proper interval bigraphs having acyclic simplification graph [1]
- PIG: Proper interval graphs
- 1-PIG: 1-Proper interval graphs [1]
- $BBHGD$: Bipartite biclique-Helly graphs with no dominated vertices [9]
- $CHBDI$: Clique independent Helly-bicovered with no dominated vertices graphs [9]
- $NSSG$: Nested separable split graphs [3, 5]

Operators and Functions

- G^2: Square of graph G
- $K(G)$: Clique graph of graph G
- $KB_m(G)$: Mutually included biclique graph of graph G [6, 7]
- $L(G)$: Line graph of graph G
- $leaves(G)$: Set of leaves (vertices of degree 1) of graph G
- $S(G)$: Simplification graph of graph G [1]

Functors

The idea behind the techniques used in most of the results on $KB(\mathcal{A})$ is to characterize $KB(\mathcal{A})$ using some other operator (or a composition of operators). That is, $KB(G) = \mathcal{F}(G)$, for $G \in \mathcal{A}$ and some operator $\mathcal{F}$.

We say that such scheme with more than one way to compute an operator is a "functor".

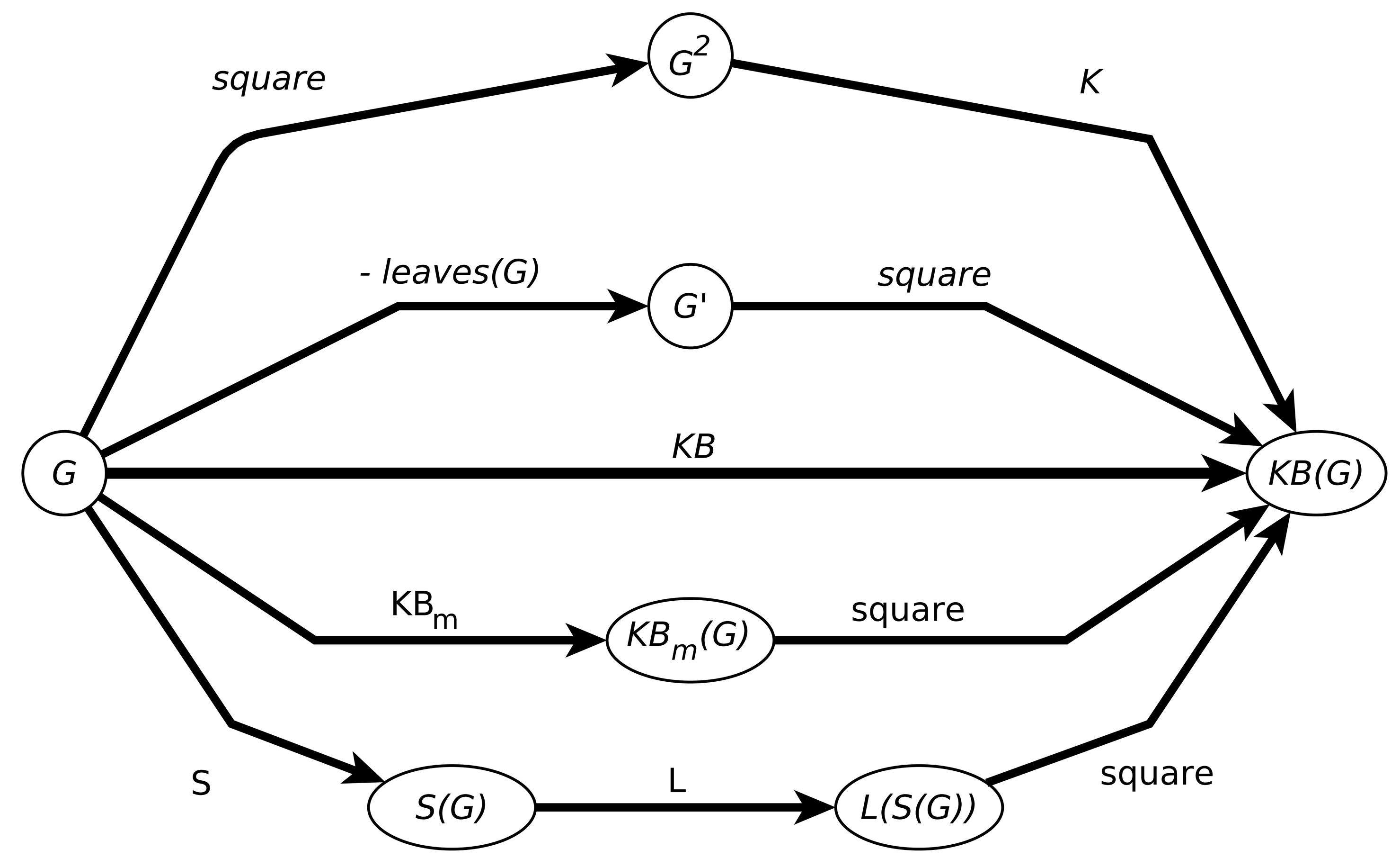

Figure 2: Summary of known functors for KB.

- $\mathbf{\{K_3, C_5, C_6\}}$**-free**: $KB(G) = K(G^2)$ [9]
  $\implies KB(\{K_3, C_5, C_6\}\text{-free}) = K((\{K_3, C_5, C_6\}\text{-free})^2)$
  Proof idea: Open neighborhood Helly, so there is a bijection between bicliques of G and cliques of G^2.
- **girth at least k** ≥ 5: $KB(G) = (G - leaves(G))^2$ [1]
  $\implies KB(\mathcal{G}_k) = (\mathcal{G}_k)^2$, with $k \geq 5$
  Proof idea: Every bicliques is a maximal star, every vertex that is not a leaf is the center of a maximal star (biclique) and to remove the leaves does not affect the girth.
- $\mathbf{K_3}$**-free**: $KB(G) = (KB_m(G))^2$ [6, 7]
  $\implies KB(K_3\text{-free}) = (KB_m(K_3\text{-free}))^2$
  Proof idea: Note that $KB_m(G) \subseteq KB(G)$ (same vertex set) and every pair of intersecting bicliques in G are at distance at most 2 in $KB_m(G)$.
- **PIB**: $KB(G) = (L(S(G)))^2$ [1]
  $\implies KB(PIB) = (L(PIB))^2$
  Proof idea: $S(PIB) = PIB$, the edges of $S(G)$ are bicliques of G, and every pair of intersecting bicliques in G are at distance at most 2 in $L(S(G))$.

Table 1 summarize the results about recognition of biclique graphs of some graph classes.

class $\mathcal{A}$	$KB(G)$**,** $G \in \mathcal{A}$	**class** $KB(\mathcal{A})$	**complexity**
complete [1]	$L(G)$	L(complete)	$\mathcal{P}$
tree [1]	$(G - leaves(G))^2$	(tree)^2	$\mathcal{P}$ (linear)
path (P_n) [1]	$\emptyset$, for $n = 1$ K_1, for $n = 2$ $(P_{n-2})^2$, for $n > 2$	(path)^2	$\mathcal{P}$ (linear)
caterpillar (tree)	$(G - leaves(G))^2$	(path)^2	$\mathcal{P}$ (linear)
cycle (C_n) [1]	K_1, for $n = 4$ $(C_n)^2$, for $n \neq 4$	(cycle)$^2 - K_4 + K_1$	$\mathcal{P}$
$\mathcal{G}_k$, for $k \geq 5$ [1] (*) [2]	$(G - leaves(G))^2$	$(\mathcal{G}_k)^2$, for $k \geq 5$	$\mathcal{P}$, for $k \geq 6$ $\mathcal{NP}$**-complete**, for $k = 5$
IBG [1, 6, 7]	**OPEN**	$\subset$ (IIC-PG)^2 $\subset K_{1,4}$-free co-CG	**OPEN**
PIB [1]	$(L(S(G))^2$	$(L(PIB))^2$	**OPEN**
PIB-ASG [1]	$(L(S(G))^2$	1-PIG	$\mathcal{P}$
HIB [9, 4]	$K(G^2)$	$\subset PIG \cap (L(PIB))^2$	**OPEN**
$\{K_3, C_5, C_6\}$-free [9]	$K(G^2)$	**OPEN**	**OPEN**
$BBHGD$ [9]	**OPEN**	$CHBDI$	**OPEN**
$NSSG$ [5]	**OPEN**	**OPEN**	$\mathcal{P}$
threshold [5]	**OPEN**	**OPEN**	$\mathcal{P}$
K_3-free [6, 7]	$(KB_m(G))^2$	$\subset \mathcal{G}^2$	**OPEN**
bipartite [6, 7]	$(KB_m(G))^2$	(IIC-comparability)^2	**OPEN**
$\mathcal{G}$ [9]	**OPEN**	Characterization	**OPEN**

Table 1: At column "$KB(G), G \in \mathcal{A}$" a brief description of $KB(G)$; at column "class $KB(\mathcal{A})$", class that is equal to (or a super-class of) $KB(\mathcal{A})$; at column "complexity", complexity (if known) of recognizing $KB(\mathcal{A})$.
(*) Note that to decide if G is the square of a graph of girth ≥ 5 is $\mathcal{NP}$-complete [2].

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