BOUNDS FOR RANGE-RELAXED GRACEFUL GAME

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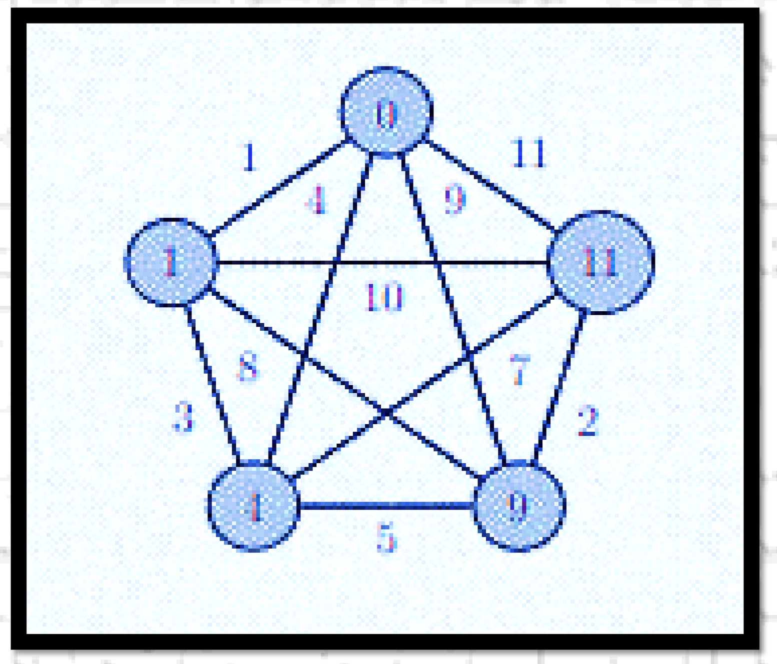
INTRODUCTION

Tuza [1] contributed to the area of graph labeling presenting many results in his seminal paper and proposing new labeling games. We investigate the Range-Relaxed Graceful game (RRG game) and present a lower bound for the number of available labels for which Alice has a winning strategy in the RRG game on a simple graph G, on a cycle and on a path graph.

RANGE RELAXED GRACEFUL LABELING

Given a graph G and the set of consecutive integer labels $L = \{0, \ldots, k\}$, $k \geq |E(G)|$, a labeling f: V(G) → L is said to be a Range-Relaxed Graceful Labeling if: (i) f is injective; (ii) each edge $uv \in E(G)$ is assigned the (induced) label $g(uv) = |f(u) - f(v)|$, then all induced edge labels are distinct.



RRG Labeling of K_5

RRG GAME

Two players, called Alice and Bob, alternately assign a previously unused label $f(v) \in L = \{0, \ldots, k\}$, $k \geq |E(G)|$ to an unlabeled vertex v of a given graph G. If both ends of uv $\in E(G)$ are already labeled, then the label of the edge is defined as $|f(u) - f(v)|$. A move is said legal if, after it, all edge labels are distinct. Alice's goal is to end up with a vertex labeling of the whole G where all of its edges have distinct labels and Bob's goal is to prevent it from happening.

OBJECTIVE

To investigate the Range-Relaxed Graceful game, present a lower bound on the number of consecutive nonnegative integer labels necessary for Alice to win the RRG game on a simple graph G and contribute to the study of the question posed by Tuza [1]:

TUZA'S QUESTION

Given a simple graph G and a set of consecutive nonnegative integer labels f(v) $\in L = \{0, \ldots, k\}$, for which values of k can Alice win the range-relaxed graceful game?

RESULTS

THEOREM 1

Let G be a simple graph on n vertices and maximum degree Δ. Alice wins the RRG game on G for any set of integer labels $L = \{0, \ldots, k\}$, with

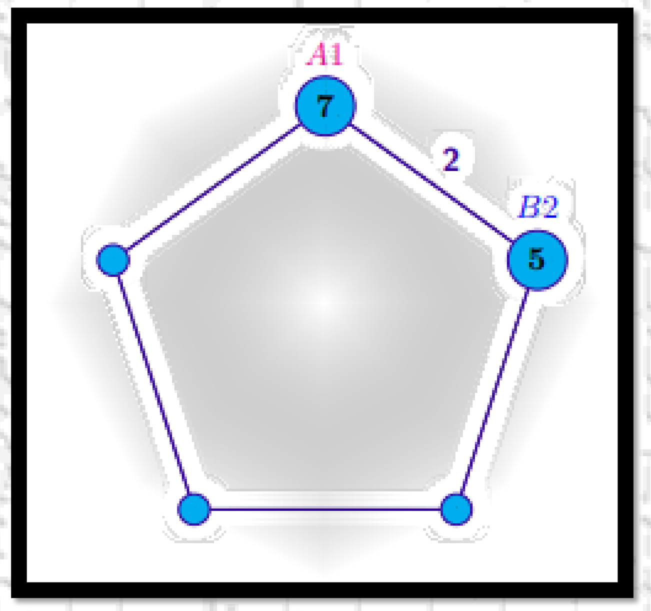
$$k \geq (2\Delta^2 + 1)(n-1) + (2\Delta + 1)\binom{n-1}{2}.$$

SKETCH OF THE PROOF

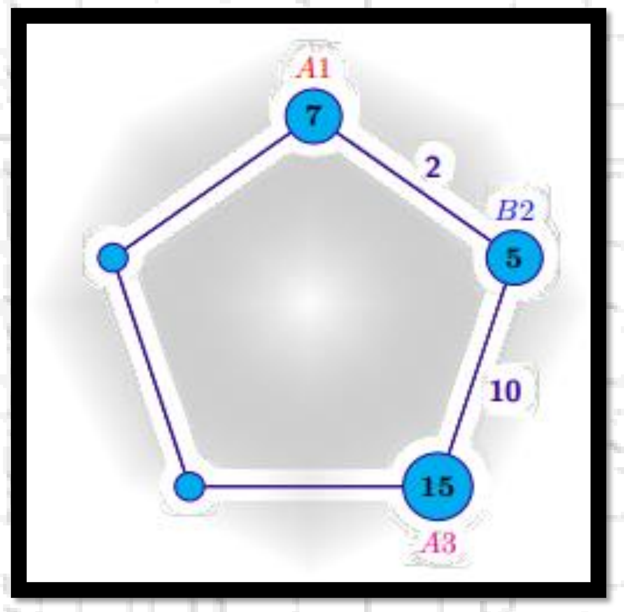
For each vertex v $\in$ V(G), we define a set of available labels L_v. When the game starts, L_v= L, for every v $\in$ V(G). At each iteration, a player assigns a label to an unlabeled vertex u from its set L_u and, then, the set of available labels of each remaining vertex is updated. Only vertex labels that can not generate repeated edge labels in future iterations can last at each set. We consider four cases that can give rise to repeated edge labels and, for each one, we count how many labels are deleted, throughout the game, from each set of available labels. From our analysis, we conclude that at most $(2\Delta^2 + 1)(n-1) + (2\Delta + 1)\binom{n-1}{2}$ labels are deleted from each set of available labels. Since |L| is greater than this value, there is always an available label at each set that can be assigned to a vertex.

EXAMPLE

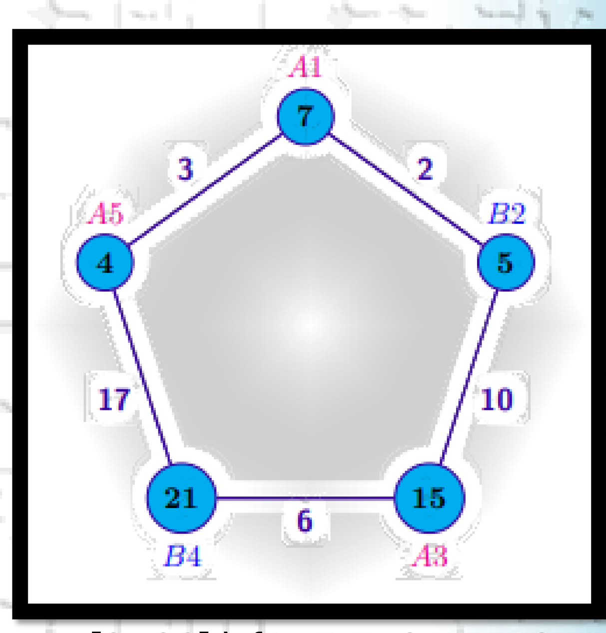
Consider C_5 and the set L = {0, 1, 2, 3, ..., 66}. Suppose that Alice starts the game by assigning label 7 to a vertex v_1. Below, we present the first three iterations, where the players play at v_1, v_2, v_3 consecutively, and we show the last iteration.



L_{v_2} = [0, 66] \ {7}
L_{v_3} = [0, 66] \ { 3, 5,7}



L_{v_4} = [0,66] \ {5,7,13,15,17,25}



L_{v_5} = [0, 66] \ {1, 5,7, 9, 11,13, 15, 17,19,21,23,27,31}

A similar proof is obtained for the following result.

THEOREM 2

Given any integer n ≥ 4, Alice wins the RRG game on the path P_n and on the cycle C_n for any set of integer labels $L = \{0, \ldots, k\}$, with $k \geq 9n - 17$.

REFERENCES

[1] Z. Tuza, Graph labeling games, Eletronic Notes in Discrete Mathematics 60 (2017), 61-68.

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